Higher-Moment Risk*

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Abstract

We estimate and analyze the ex ante higher order moments of stock market returns. We document that even and odd higher-order moments are strongly negatively correlated, creating periods where the return distribution is riskier because it is more left-skewed and fat tailed. Such higher-moment risk is negatively correlated with variance and past returns, meaning that it peaks during calm periods. The variation in higher-moment risk is large and causes the probability of a two-sigma loss on the market portfolio to vary from 3.3% to 11% percent over the sample, peaking in calm periods such as just before the onset of the financial crisis. In addition, we argue that an increase in higher-moment risk works as an "uncertainty shock" that deters firms from investing. Consistent with this argument, more higher-moment risk predicts lower future industrial production.

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Uncertainty about future economic outcomes is a key driver of asset prices and economic fluctuations more generally: everything else equal, higher uncertainty depresses asset prices and makes investments less attractive.[1] In this regard, investors care not only about the variance of outcomes but also about the higher order moments of the distribution of outcomes, and in particular whether very bad outcomes are relatively more likely. To understand the role of such higher-moment risk, we estimate the higher order moments of the return distribution for the U.S. market portfolio and establish new stylized facts that have broad implications for investors, asset prices, and economic fluctuations more generally.

We base our analysis on ex ante moments that are estimated from options prices. We estimate the moments of the risk-neutral distribution of stock returns, which is the physical distribution adjusted for state prices. Using methods based on Martin (2017), we translate these risk-neutral moments into physical moments as perceived by an unconstrained power utility investor who wants to hold the market portfolio. These moments are entirely forward looking and, unlike risk-neutral moments, contain no adjustment for risk, which makes them well suited for studying time-variation in higher-moment risk.

We first study how the higher order moments of the return distribution vary over time. We find that the third and the fifth moments (skewness and hyperskewness) are highly positively correlated, and similarly that the fourth and the sixth moments (kurtosis and hyperkurtosis) are highly positively correlated. More importantly, the skewness related moments are negatively correlated with the kurtosis related moments, meaning that there are periods where the return distribution becomes both more left-skewed (due to large negative odd number moments) and more fat tailed (due to large positive even number moments). This comovement in higher order moments is so strong that the first principal component of the space spanned by skewness, kurtosis, hyperskewness, and hyperkurtosis explains 91% of the joint variation in higher order moments.

We define the first principal component of the higher order moments as the higher-moment risk index. Indeed, the first principal component eigenvector has a negative sign for skewness and hyperskewness, and a positive sign for kurtosis and hyperkurtosis. As shown in Ebert (2013), an investor with power utility has preference for odd

[1]For asset prices, see Sharpe (1964); Lintner (1965); Black (1972); for investments, see Tobin (1969), Cochrane (1991), and Bloom (2009).

number moments of any order and is averse to even number moments of any order. A high value of the first principal component can therefore be interpreted as times when higher order moment risks are high in the sense that the moments are less favorable for a power utility investor (the distrubtion is more left-skewed and has fatter tails).

The higher-moment risk index varies substantially over time with the state of the financial markets. The index is negatively correlated with the ex ante variance, or second moment, of the return distribution with a correlation of -0.46, and it is positively correlated with past two-year returns with a correlation of 0.22. Accordingly, higher-moment risk peaks during seemingly calm periods where variance is low and past returns are high. Or vice versa, risk does not disappear during calm periods, it hides in the tail of the return distribution.

This time variation in higher moment risk has important implications for financial markets and investors. Consider for instance the probability that realized market returns are two standard deviations below expected returns, a tail risk that is driven entirely by higher order moments and would be a constant 2.5% if the returns on the market portfolio followed a normal distribution. We derive this probability explicitly from option prices and find that it varies from 3.3% to 11% over the sample, implying a 333% variation in tail risk over the sample. In addition, because variance is negatively correlated with higher-moment risk, the tail risk peaks at periods where the variance is low. These results are particularly important for sophisticated investors such as hedge funds, who manage large notionals in constant-volatility portfolios. These portfolios are scaled to have a constant variance over time with the aim of keeping portfolio risk constant. However, because higher-moment risk varies over time, the risk in the constant-volatility portfolios varies, peaking in seemingly calm periods such as just before the onset of the financial crisis. To the extend that these sophisticated investors have a systematic influence on asset markets,[2] this tendency to systematically load on tail risk following market run ups may be worrying for regulators.

Going beyond the financial markets, higher-moment risk also have potential implications for the real economy. Indeed, a large literature starting with Bloom (2009) argues that uncertainty about the economy depresses investment because it increases the option value of postponing it: when uncertainty is high, firms prefer to see which state of the economy materializes before they make an investment. Similarly, when

[2]See He and Krishnamurthy (2013); He, Kelly, and Manela (2016) Muir, and other for evidence on the effect of levered investors on asset prices and the economy.

higher-moment risk is high, firms may prefer to delay investment until after the risk of a crash diminishes.

We find empirical evidence consistent with the hypothesis that higher-moment risk deters investment. We find empirically that a higher probability of a two-sigma loss predicts lower future industrial production. The effect is stronger the longer the horizon. For the three year horizon, the probability of a two-sigma event predicts future industrial production with an R^2 of as much as 22 percent. In addition, the effect of higher-moment risk on industrial production is substantially stronger than the effect of political uncertainty as measured by Baker, Bloom, and Davis (2016), which is weakly correlated with future industrial production and only at the very short horizon, raising questions about whether high uncertainty causes low growth, or low growth causes high uncertainty, as argued by Berger, Dew-Becker, and Giglio (2017). In contrast, higher-moment risk predicts industrial output even stronger if we skip one year and predict growth in industrial production in year two and three, lending confidence to the idea that higher-moment risk causes low growth, and not the other way around.

As the final part of the paper, we study why higher-moment risk varies over time and tend to be high in calm periods. First, we note that our results are closely related to Bunnermeier and Sannikov (2014). Brunnermeier and Sannkikov model a macro economy with a financial sector and find that the risk in the economy is highest when volatility is low, which they dub the volatility paradox. The volatility paradox arises because times with low volatility are times where the financial sector is the most levered and thus most sensitive to a negative funding liquidity shock that causes sophisticated investors to de-lever and potentially induce a firesale. To test if such dynamics may drive our results, we test whether intermediary leverage and liquidity explains variation in the higher-moment risk index. We find no relation between higher-moment risk and intermediary leverage as measured by He, Kelly, and Manela (2016), but we note that this lack of correlation might arise from the fact that their measure is counter- rather than pro-cyclical, peaking in crisis where the equity value is low. However, we find that higher-moment risk is positively related to both market and funding liquidity as measured by the bid-ask spread and the ted spread, suggesting that the availability to fund and trade investments may be an important driver of higher-moment risk.

We also investigate if higher-moment risks may be related to "bubble" characteristics and market valuation. Indeed, one could interpret that fact that higher-moment risk is high following run ups as though investors fear that the run ups have resulted in bubbles that may burst. We therefore test if variation in higher-moment risk can be explained by the following "bubble" characteristics suggested in the literature: acceleration (Greenwood, Shleifer, and You (2017)), turnover (Chen, Hong, and Stein (2001)), and issuance percentage (Pontiff and Woodgate (2008)). We do not find strong evidence that higher-moment risk is related to any of the bubble characteristics. We do find, however, that higher-moment risk is negatively correlated with the transitory component of consumption to wealth (i.e. cay from Lettau and Ludvigson, 2001), suggesting that it peaks during periods of high wealth relative to consumption.

Our paper relates to and extends the existing literature on estimating time-varying market tail risk by integrating two different approaches. Previous research on tail risk is based on either (1) physical moments based on backward looking information or (2) risk-neutral moments based on forward looking option prices. We show that physical higher-moment risks can be estimated in a forward looking manner, and in real time, which complements the existing literature that uses historical (backward looking) returns to estimate tail risks, such as Bollerslev and Todorov (2011) who estimate tail risk using high frequency intraday returns and Kelly and Jiang (2014) who estimate market wide tail risks from the cross-section of firm-level returns. Our paper also relates to the literature that studies tail risk using option prices. This existing literature studies risk-neutral moments and probabilities (e.g. Siriwardane (2015), Gao, Gao, and Song (2017), Gao, Lu, and Song (2017), Bates (2000), and Schneider and Trojani (2017)), whereas we study physical moments and probabilities.

In summary, the physical higher order moments of the return distribution can be measured in real time. The higher order moments comove strongly, creating periods where the higher-moment risk is high in the sense that the distribution is more left skewed and fat tailed. A single factor explains 91% of the joint variation in higher order moments. The higher-moment risk is high in periods that appear calm as measured by ex ante volatility and past returns. This variation in higher-moment risk is broadly consistent with the model of a macro economy with a levered financial sector in Brunnermeier and Sannikov (2014). In addition, these stylized facts about higher-moment risk has strong implications for investors as it causes the tail risk of a constant-volatility portfolio to vary by more than 300%. Finally, more higher-moment risk predicts lower future industrial production, consistent with the risk of a crash deterring firms from investing.

The paper proceeds as follows: Section 1 covers the theory behind how we estimate higher order moments and tail probabilities. Section 2 covers the data and the empirical implementation. Section 3 and 4 studies time variation in higher-moment risks. Section 5 discusses robustness of our results on time variation in higher-moment risk. Section 6 studies the implications of time-varying higher-moment risks for investors. Section 7 studies the relation between higher-moment risk and future industrial production. Section 8 studies the economic drivers of higher-moment risks. Section 9 concludes the paper.

1 Inferring Ex Ante Moments from Asset Prices

We consider an economy where agents can trade two assets, a risk-free asset and a risky asset. The risk-free asset earns a gross risk-free rate of return $R^f_{t,T}$ between time t and time T. The risky asset has a price of S and earns a random gross return $R_{t,T}$. The risky asset pays dividends, $D_{t,T}$, between time t and time T such that its gross return is $R_{t,T} = (S_T + D_{t,T})/S_t$.

Starting from the standard asset pricing formula, we can relate risk-neutral and physical expected values of the time T random payoff, X_T, as

$$E_t[X_T m_{t,T}] = E^*_t[X_T]/R^f_{t,T} \tag{1}$$

where the asterisk denotes risk-neutral expectation and $m_{t,T}$ is a stochastic discount factor. If we define the time T random payoff, $X_{t,T}(n)$, in the following way

$$X_{t,T}(n) = R^n_{t,T} m^{-1}_{t,T} \tag{2}$$

then equation (1) implies that the n'th moment of the risky asset's physical return distribution can be expressed in terms of the risk-neutral expectation of $X_{t,T}(n)$:

$$E_t[R^n_{t,T}] = E_t[\underbrace{R^n_{t,T} m^{-1}_{t,T}}_{X_{t,T}(n)} m_{t,T}] = E^*_t[\underbrace{R^n_{t,T} m^{-1}_{t,T}}_{X_{t,T}(n)}]/R^f_{t,T} \tag{3}$$

So if we know the pricing kernel m, then we can derive all moments of $R_{t,T}$ directly from risk-neutral pricing of the claim to $X_{t,T}(n)$. Following Martin (2017), we compute the physical expected value of $R^n_{t,T}$ from the point of view of an unconstrained rational power-utility investor who chooses to be fully invested in the market. This investor has initial wealth W_0 and terminal wealth $W_T = W_0 R_{t,T}$. Given the investor's utility function, $U(x) = x^{1-\gamma}/(1-\gamma)$, with relative risk-aversion, γ, we can determine the investor's stochastic discount factor. Specifically, combining the first order condition from the investor's portfolio choice problem with the fact that the investor holds the market, the stochastic discount factor becomes proportional to $R^{-\gamma}_{t,T}$:

$$m_{t,T} = kR^{-\gamma}_{t,T} \tag{4}$$

for some constant k which is unobservable to us. However, we do not need to learn k to estimate physical moments; we can correct for k by rewriting (3) in the following way. First, setting $n = 0$ in (2) we get $X_{t,T}(0) = m^{-1}_{t,T}$ and the standard asset pricing formula (1) then implies the relation:

$$E^*_t[m^{-1}_{t,T}] = R^f_{t,T} \tag{5}$$

Then, inserting (5) and (4) into (3), we obtain an expression of the n'th physical moment perceived by an unconstrained rational power utility investor who chooses to be fully invested in the market:

$$E_t[R^n_{t,T}] = \frac{E^*_t[R^n_{t,T}\overbrace{R^{\gamma}_{t,T}/k}^{m^{-1}_{t,T}}]}{E^*_t[\underbrace{R^{\gamma}_{t,T}/k}_{m^{-1}_{t,T}}]} = \frac{E^*_t[R^{n+\gamma}_{t,T}]}{E^*_t[R^{\gamma}_{t,T}]} \tag{6}$$

since k is a constant.

The relation between physical and risk-neutral moments shown in (6) is central to our empirical analysis. The key insight is that we can estimate the n'th physical moment directly from risk-neutral pricing of $R^{\gamma}_{t,T}$ and $R^{n+\gamma}_{t,T}$. Furthermore, by pricing claims to the payoffs $R^{m+\gamma}_{t,T}$ for $m \in \{1, ..., n\}$, we can then estimate *standardized* moments.

To understand how we estimate standardized moments from (6), recall the notion of the n'th standardized moment formula:

$$n\text{'th standardized moment of } R_{t,T} = E_t\left[\left(\frac{R_{t,T} - E_t[R_{t,T}]}{\text{Var}[R_{t,T}]^{1/2}}\right)^n\right] \tag{7}$$

Expanding (7) and replacing physical moments with risk-neutral counterparts as presented in equation (6), we can arrive at expressions for all physical standardized moments as functions of risk-neutral moments. For example, the third standardized physical moment (skewness) can be expressed in terms of risk-neutral moments by first expanding (7) with $n = 3$:

$$\text{Skewness}_{t,T} = \frac{E_t[R_{t,T}^3] - 3E_t[R_{t,T}]E_t[R_{t,T}^2] + 2E_t[R_{t,T}]^3}{(E_t[R_{t,T}^2] - E_t[R_{t,T}]^2)^{3/2}} \tag{8}$$

and then replacing the physical moments in (8) with the risk-neutral counterparts using equation (6). Similar expressions can be written up for other higher order moments of interest, as seen in Appendix A. Importantly, the right-hand-side of (6) consists of asset prices which can be estimated directly from current and observable call and put options written on the risky asset. Hence, higher order moments can be estimated in real time, without using historical realized returns or accounting data.

1.1 Inferring Ex Ante Market Tail Probabilities

Next, we show how we estimate ex ante tail probabilities from option prices written on the market. To understand our approach, note first that the probability at time t of a market return that is lower than α at time T can be written as the physical expectation of an indicator function in the following way

$$P_t(R_{t,T} < \alpha) = E_t[1_{\{R_{t,T}<\alpha\}}] \tag{9}$$

Using the standard asset pricing formula in (1), we can rewrite the probability in terms of the risk-neutral measure by adjusting the right hand side of equation (9) for the inverse of the stochastic discount factor in (4)

$$P_t(R_{t,T} < \alpha) = \frac{E_t^*[R_{t,T}^\gamma 1_{\{R_{t,T}<\alpha\}}]}{E_t^*[R_{t,T}^\gamma]} \tag{10}$$

The right hand side of (10) is an asset price that has the simple representation presented in Proposition 1, which generalizes Result 2 in Martin (2017) from log-utility to general power utility for any level of relative risk-aversion.

Proposition 1 *For the unconstrained rational power utility investor who wants to hold the market, the conditional physical probability that market return from time t to T is lower than α is:*

$$P_t(R_{t,T} < \alpha) = \frac{R^f_{t,T}}{E^*_t[R^\gamma_{t,T}]}\Bigg[\alpha^\gamma put'_{t,T}(\alpha S_t - D_{t,T}) - \frac{\gamma}{S_t}\alpha^{\gamma-1} put_{t,T}(\alpha S_t - D_{t,T}) \\ + \int_0^{\alpha S_t - D_{t,T}} \frac{\gamma(\gamma-1)}{S_t^2}\left(\frac{K + D_{t,T}}{S_t}\right)^{\gamma-2} put_{t,T}(K)dK\Bigg]$$

where $put'_{t,T}(\alpha S_t - D_{t,T})$ is the first derivative of the put option price with strike $\alpha S_t - D_{t,T}$.

Proof. The results of Breeden and Litzenberger (1978) imply the equality

$$E^*_t[R^\gamma_{t,T} 1_{\{R_{t,T}<\alpha\}}] = R^f_{t,T} \int_0^\infty \left(\frac{K + D_{t,T}}{S_t}\right)^\gamma 1_{\{K<\alpha S_t - D_{t,T}\}} \mathrm{put}''_{t,T}(K)dK$$

where $\mathrm{put}''_{t,T}(K)$ is the second derivative of the put option price written on the underlying process S. Splitting the integral at $\alpha S_t - D_{t,T}$ we have

$$E^*_t[R^\gamma_{t,T} 1_{\{R_{t,T}<\alpha\}}] = R^f_{t,T} \int_0^{\alpha S_t - D_{t,T}} \left(\frac{K + D_{t,T}}{S_t}\right)^\gamma \mathrm{put}''_{t,T}(K)dK$$

Proposition 1 then follows from using integration by parts twice. ■

2 Data and Empirical Implementation

We use the Ivy DB database from OptionMetrics to extract information on vanilla call and put options written on the S&P 500 index for the last trading day of every month. The data is from January 1996 to December 2017. We obtain implied volatilities, strikes, closing bid-prices, closing ask-prices, and maturities. As a proxy for the risk-free rate, we use the zero-coupon yield curve from the Ivy DB database, which is derived from the LIBOR rates and settlement prices of CME Eurodollar futures. We also obtain expected dividend payments. We consider options with times to maturity between 10 and 360 calender days, and apply standard filters, excluding options with implied volatility higher than 100%.

2.1 Estimating Market Moments

There is a large body of literature devoted to pricing asset derivatives such as those in (6), using observable option prices written on the asset. Indeed, Breeden and Litzenberger (1978), Bakshi and Madan (2000), and Bakshi, Kapadia, and Madan (2003) show that the arbitrage free price of a claim on some future (twice differentiable) payoff can be expressed in terms of a continuum of put and call option prices. Specifically for our purposes, using the results of Breeden and Litzenberger (1978), Martin (2017) shows that we can write the n'th physical moment of $R_{t,T}$ as

$$E_t[R_{t,T}^n] = \frac{E_t^*[R_{t,T}^{n+\gamma}]}{E_t^*[R_{t,T}^{\gamma}]} = \frac{(R_{t,T}^f)^{n+\gamma} + R_{t,T}^f\,[p(n+\gamma) + c(n+\gamma)]}{(R_{t,T}^f)^{\gamma} + R_{t,T}^f\,[p(\gamma) + c(\gamma)]} \tag{11}$$

with

$$\begin{aligned} p(\theta) &= \int_0^{F_{t,T}} \frac{\theta(\theta-1)}{S_t^\theta} \left(S_t R_{t,T}^f - F_{t,T} + K\right)^{\theta-2} \mathrm{put}_{t,T}(K)dK \\ c(\theta) &= \int_{F_{t,T}}^{\infty} \frac{\theta(\theta-1)}{S_t^\theta} \left(S_t R_{t,T}^f - F_{t,T} + K\right)^{\theta-2} \mathrm{call}_{t,T}(K)dK \end{aligned} \tag{12}$$

where $F_{t,T}$ is the forward price and $\mathrm{call}_{t,T}(K)$ and $\mathrm{put}_{t,T}(K)$ are call and put option prices written on the risky asset at time t with horizon $T-t$ and strike K.

In practice, we do not observe a continuum of call and put options and therefore (11) must be numerically approximated. To see how we approach this numerical approximation, let $F_{t,T}$ be the forward price and, then using the notation from Martin (2017), we can write the price, $\Omega_{t,T}(K)$, at time t of an out-of-the money option with strike K and maturity T as

$$\Omega_{t,T}(K) = \begin{cases} \mathrm{call}_{t,T}(K) & \text{if } K \geq F_{t,T} \\ \mathrm{put}_{t,T}(K) & \text{if } K < F_{t,T} \end{cases}$$

We let $K_1, ..., K_N$ be the (increasing) sequence of observable strikes for the N out-of-the money put and call options and define $\Delta K_i = \frac{K_{i+1}-K_{i-1}}{2}$ with

$$\Delta K_i = \begin{cases} K_{i+1} - K_i & \text{if } i = 1 \\ 2K_i - K_{i-1} & \text{if } i = N. \end{cases}$$

We approximate the integrals in (12) by observable sums such that the n'th physical moment becomes:

$$E_t[R_{t,T}^n] = \frac{(R_{t,T}^f)^{n+\gamma} + R_{t,T}^f \left[\sum_{i=1}^N \frac{(n+\gamma)(n+\gamma-1)}{S^{n+\gamma}} (S_t R_{t,T}^f - F_{t,T} + K_i)^{n+\gamma-2} \Omega_{t,T}(K_i) \Delta K_i\right]}{(R_{t,T}^f)^{\gamma} + R_{t,T}^f \left[\sum_{i=1}^N \frac{\gamma(\gamma-1)}{S^{\gamma}} (S_t R_{t,T}^f - F_{t,T} + K_i)^{\gamma-2} \Omega_{t,T}(K_i) \Delta K_i\right]} \tag{13}$$

In summary, combining equation (13) with the standardized moment formula in equation (7), we can express standardized physical moments in terms of the derivatives prices written on the risky asset.

When we estimate physical moments for a given horizon, say T, for which we do not observe put and call prices, we linearly interpolate the (standardized) moments between the two closest horizons available in the data. In a few cases, we need to extrapolate to obtain moments for the desired horizon.

2.2 Estimating Market Tail Probabilities

The main challenge when implementing Proposition 1 is that we are required to estimate the first derivative of the put option price written on the risky asset at strike $\alpha S_t - D_{t,T}$. To handle a sparse and discrete set of observed option prices, we smoothen observed option prices using a Gaussian kernel smoothening procedure. Specifically, we smoothen implied volatilities around the strike $\alpha S_t - D_{t,T}$ and choose the kernel bandwidth to minimize the squared errors between the observed and estimated implied volatilities under the constraint that the estimated option prices do not allow for arbitrage.

Given a smooth set of option prices around the strike $\alpha S_t - D_{t,T}$, we compute the first derivative as the slope between the two adjacent prices:

$$\text{put}'_{t,T}(\alpha S_t - D_{t,T}) = \frac{\text{put}_{t,T}(\alpha S_t - D_{t,T} + h) - \text{put}_{t,T}(\alpha S_t - D_{t,T} - h)}{2h} \tag{14}$$

where h is the chosen grid step size in the discretization.

Let $K_1, ..., K_M$ be the (increasing) sequence of observable strikes for the M out-of-the money put options where K_M is the observed strike that is closest to $\alpha S_t - D_{t,T}$. We approximate the integral in Proposition 1 by the observable sum:

$$\sum_{i=1}^{M} \frac{\gamma(\gamma-1)}{S_t^2} \left(\frac{K_i + D_{t,T}}{S_t}\right)^{\gamma-2} \mathrm{put}_{t,T}(K_i)\Delta K_i \tag{15}$$

Inserting (14) and (15) into Proposition 1, we can estimate physical probabilities.

2.3 Determining Market Risk-Aversion

To estimate physical moments and tail probabilities, we have to choose a level of risk-aversion. Following Bliss and Panigirtzoglou (2004), we use the Berkowitz (2001) test to estimate the constant level of risk aversion that best reconciles the cross-section of options prices with the time-series realizations of the S&P 500 index.[3] Doing so, we find that a $\gamma = 3$ is the best fit to monthly returns in our sample, and we therefore rely of this level of risk aversion in our main analysis. Our estimate is close to the $\gamma = 4$ that Bliss and Panigirtzoglou (2004) find best explains the 1983 to 2001 sample.

Tabel 1 reports moment summary statistics for moments estimated with risk aversion of 0 (risk-neutral investor), 1 (log-utility), 3 (our baseline case), and 5. The average ex ante estimated skewness is negative at both monthly and quarterly horizons and all levels of risk aversion, suggesting that the physical distributions are left skewed. Consistent with the results of Neuberger (2012), we find that average skewness is not diminishing in the horizon, in the sense that skewness is close to the same on a monthly and quarterly horizon. In addition, average kurtosis is larger than 3 for

[3]The idea behind the Berkowitz test is that, for the true value γ, the distribution of $u_{t,T} = F_{t,T}(R_{t,T})$ is uniform and the distribution $y_{t,T} = \Phi^{-1}(u_{t,T})$ is standard normal. Here $F_{t,T}(R_{t,T})$ denotes the distribution function

$$F_{t,T}(r) = \int_{-\infty}^{r} \frac{\pi_{t,T}(x)x^{\gamma}}{\int_{-\infty}^{\infty} \pi_{t,T}(y)y^{\gamma}dy} dx$$

where $\int_{-\infty}^{\infty} \pi_{t,T}(y)y^{\gamma}dy$ is simply a normalization constant which ensures that the physical return distribution integrates to one. In the Berkowitz test, we estimate the coefficients in the regression model:

$$y_{t,T} = \hat{a} + \hat{\beta} y_{t-1,T-1} + \epsilon_{t,T}, \qquad \epsilon_{t,T} \sim N(0, \hat{\sigma})$$

and perform a likelihood ratio test of the joint hypothesis that $a = \beta = 0$ and $\mathrm{Var}(\epsilon_{t,T}) = 1$.

both horizons and all levels of risk aversion, suggesting that the physical distributions are leptokurtic (i.e. tails are one average fatter than the normal distribution). We report in later analysis that our results are not sensitive to the choice of $\gamma = 3$ as our baseline case.

3 Time Variation and Comovements in Higher Order Moments

In this section, we study time variation and comovements in higher order moments. We consider the first six moments of the market return distribution. The third and the fifth moments capture the skewness of the distribution, and the fourth and the sixth moments capture the kurtosis of the distribution.

Figure 1 and Figure 2 plot the time series of the skewness, kurtosis, hyperskewness, and hyperkurtosis of the return distribution at the monthly and quarterly horizon. The skewness moments trend down during the sample and the kurtosis moments trend up, suggesting that higher-moment risk increases during the sample; at monthly horizon in particular, hyperkurtosis and hyperskewness exhibit a dramatic drop/rise during the end of the sample. The results presented throughout the paper are not driven by this latest dramatic rise in higher-moment risk – in fact, all our results are stronger if we exclude the last two years. In addition, the moments also exhibit substantial variation over the sample. Consider for instance skewness and kurtosis at the quarterly horizon in Figure 2. During the period from 2003 until 2007, skewness dropped substantially, meaning the distribution became more left skewed. In addition, kurtosis increased during this period, suggesting that the riskiness of the entire distribution increased, and highlighting that there is strong comovements in higher order moments.

Table 2 reports the monthly (Panel A) and quarterly (Panel B) pairwise correlations between the moments. The green (lower right) box shows pairwise correlations between higher order moments. Since investors are averse to lower skewness and hyperskewness (i.e. averse to a more left-skewed distribution), we have flipped the signs of these moments such that we can interpret a higher value of the third and fifth moments as higher risk. The correlations between the higher order moments are all positive and large, suggesting that the risk in the individual higher order moments tend to move together.

The strong comovements between higher order moments suggests that the joint variation in higher order moments can be attributed to a single factor. We therefore estimate the principal components of the space spanned by skewness, kurtosis, hyperskewness, and hyperkurtosis. The four principal components are shown in Table 3. At both the monthly and quarterly horizon, the first principal component explains about 91% of the joint variation in higher order moments, underlining the strong co-movement in higher-moment risks.

The first principal component eigenvectors have the same signs for skewness and hyperskewness, while the sign is opposite for kurtosis and hyperkurtosis. We standardize each moment to make the eigenvector loadings comparable. The size of the loadings for the first principal components are very similar across the moments, namely -0.46 (-0.47 quarterly) for skewness, 0.52 (0.52 quarterly) for kurtosis, -0.52 (-0.52 quarterly) for hyperskewness, and 0.50 (0.50 quarterly) for hyperkurtosis, implying that the first principal component is approximately the average of the standardized higher order moments with the signs flipped for skewness and hyperskewness. As shown in Ebert (2013), an investor with power utility has a preference for odd number moments of any order and is averse to even number moments of any order. A high value of the first principal component can therefore be interpreted as times when higher order moments (the moments that add mass to the lower tail of the return distribution) are on average large. Accordingly, we define the first principal component as a higher-moment risk index.

Higher-Moment Risk Index: *We define a higher-moment risk index (HRI) as the first principal component of the space spanned by skewness, kurtosis, hyperskewness, and hyperkurtosis.*

4 Systematic Variation in the Higher-Moment Risk Index

We next first study how the higher-moment risk index relates to the second moment of the return distribution (i.e. variance). Figure 3 displays the time-series plot of the monthly HRI along with the variance. The two times series are negatively correlated: higher-moment risk is high when variance is low. For instance, HRI is low during the burst of the tech bubble and during the financial crisis, where variance is somewhat high, and HRI is high during the low variance period from 2004 to 2007 and again from 2012 an onwards where variance is low. The higher-moment risk spikes during 2017 where volatility is historically low.

This negative relation between variance and higher-moment risk is highly statistically significant as shown in Panel A of Table 4. On a monthly horizon, the magnitude of the correlation between variance and HRI is -0.46 with 95% bootstrapped confidence bounds of $[-0.53, -0.42]$. On the quarterly horizon, the correlation is -0.59 with 95% confidence bounds of $[-0.65, -0.56]$. All the higher-order moment contribute to the negative correlation between HRI and variance. Indeed, the blue (upper right) box of Panel A of Table 2 shows the pairwise correlations between variance and higher order moments. Variance is negatively correlated to the negative of skewness, kurtosis, the negative of hyperskewness, and hyperkurtosis with correlations ranging from -0.53 to -0.34.

We next study how higher-moment risk relates to past returns. Figure 4 shows time-series plots of the past two year return and the HRI. The past returns and the HRI are positively correlated with correlations of 0.22 both on the monthly and quarterly horizons. Panel B of Table 4 reports the bootstrapped 95% confidence bounds for the correlations between the HRI and past returns. The confidence bounds are [0.14, 0.31] on the monthly horizon and [0.13, 0.31] on the quarterly horizon.

Panel C of Table 4 shows the pairwise correlations between past returns and the individual physical moments. We find a negative relation between past returns and variance. This finding is consistent with the intuition that times after market run-ups are calm times where variance is low. Looking at skewness, we find a negative relation with past returns, implying that the return distribution tilts to the right and leaves more probability mass in the left tail of the return distribution subsequent to market run-ups. Similarly, kurtosis is positive in past returns, hyperskewness is negative in past returns, and hyperkurtosis is positive in past returns. The results are quantitatively similar for monthly and quarterly moments.

5 Robustness of the Higher-Moment Risk Index

We conduct several robustness analyses to solidify our results on the higher-moment risk index. First, a potential concern is that option liquidity is low during periods of financial market turbulence which could be reflected in large option bid-ask spreads during such periods. Such fluctuations in bid-ask spreads could pose a problem when we use mid prices to estimate our higher-moment risk index. Figure 5 shows the higher-moment risk index when using exclusively bid, mid, or ask prices. As seen from the figure, the higher-moment risk index is almost identical when using either bid, mid, or ask prices. The pairwise correlations between the higher-moment risk indexes implied by bid or ask prices with the mid price higher-moment risk index are 0.996 and 0.998 respectively, suggesting that our results are not driven by time-varying bid-ask spreads on option prices.

A second concern is that our results are sensitive to our choice of a constant risk-aversion coefficient equal to three ($\gamma = 3$). Table 10 shows the pairwise correlations between risk-neutral higher order moments. Interestingly, the comovements between risk-neutral higher order moments are similar to those between physical higher order moments. That is, the green (lower right) box of Table 10 shows the pairwise correlations between the negative of (risk neutral) skewness, kurtosis, the negative of hyperskewness, and hyperkurtosis. These correlations are all positive and close to one, suggesting that also under the risk-neutral measure there is a common component in the higher order moments. Table 11 shows the principal components of the space spanned by *risk-neutral* skewness, kurtosis, hyperskewness, and hyperkurtosis. The first principal component explains 95% of the joint variation in risk-neutral higher order moments. The correlation between this first principal component and the first principal component of the space spanned by *physical* skewness, kurtosis, hyperskewness, and hyperkurtosis is 0.97.

Finally, one may worry that the time variation in the higher-moment risk index is driven by time variation in risk aversion. However, variation in risk aversion is likely to increase the variation in higher order moments and strengthen our results. To see this, note that the risk neutral distribution is more left skewed than the physical distribution because the high state prices on the poor outcomes of the market increase the mass in the left tail of the risk neutral distribution. The lower the risk averision, the closer are the physical and risk neutral distributions. When risk averision is low, the physical distribution thus moves towards the risk neutral distribution and become more left skewed (for any given risk neutral distribution). Accordingly, time varying risk aversion would cause higher-moment risk to be higher than our estimate when risk aversion is low, such as in calm times, and it would cause higher-moment risk to be lower than our estimate when risk aversion is high, such as during the financial crisis. In other words, time variation in risk aversion would likely strengthen the pattern of the time variation in higher-moment risk that we document.

6 Implications for Investors

We next analyze the implications of our new stylized facts about higher-moment risk for investors. We first study how higher-moment risk influences the probability of a tail event on the market portfolio. We define the tail risk as the probability that the realized market return is more than two or three standard deviations below the expected value. We consider tail risk probabilities relative to both the conditional and unconditional standard deviation of the market return. Variation in the tail risk relative to the conditional standard deviation is entirely driven by higher order moments because it is measured relative to the conditional expected return and variance (i.e. the first two moments). One the other hand, the tail risk relative to the unconditional standard deviation is also influenced by the conditional variance, as it is measured relative to the conditional expected return and the unconditional variance.

We derive the probabilities of tail events using Proposition 1 and our estimated moments. Figure 6 Panel A plots the time series of the probability that the market return is two conditional standard deviations below the expected returns. The solid blue line is the tail risk observed from option prices. This tail risk peaked on June 31^{th} 2017 with a probability of 11%, more than three times the size of its low on June 31^{th} 1997, where the probability was 3.3%. The probability was also high before the financial crisis.

The horizontal dotted line is the probability implied by a normal distribution. The line is flat, reflecting that the risk of a two-sigma event is constant for a normal distrubtion. The grey area between the two lines is higher moment risk: it is the additional risk of a tail event that arises from the higher order moments of the return distribution. This probability of a tail loss in excess of what is implied by a normal distribution ranges from 0.8% to 8.5%, showing that higher order moments constitute most of the tail risk for investors and drives the time variation.

Panel B of Figure 6 plots the time series of the probability that the market return is two *unconditional* standard deviations below the expected returns. Two unconditional standard deviations is approximately 10%. The solid blue line is the tail risk observed from option prices and the dotted line is the probability implied by a normal distribution. The shaded area between the two is again higher moment risk. The figure shows that most of the variation in the probability of a 10% loss is captured by the normal distribution, i.e. it is captured by conditional volatility. However, there is still substantial variation coming from higher moment risk, and in many parts of the sample, higher order moments constitute the majority of the probability of the 10% percent drop.

Figure 8 shows similar results for the probability of a three-sigma loss on the market portfolio. The probability that realized returns are more than three conditional standard deviation below expected return is shown in Panel A. It peaked on June 31st 2017 with a probability of 4.2%, which is five and a half times the size of its low on February 27th 2008, where the probability was 0.8%. These probabilities are far from what is implied by a normal distribution, which is 0.13%. Specifically, the average probability of a $-3\sigma_{t,t+1}$ event is 1.8%, which is fourteen times higher than what is implied by the normal distribution.

Figure 7 and Figure 9 shows the same probabilities at the quarterly rather than monthly horizon. The results are similar, except that the importance of higher-moment risk for tail loss probabilities is larger than at the monthly horizon.

To make the analysis more tangible and to ease the interpretation, we relate the tail risk probabilities to two different investors. The tail risk relative to the unconditional standard deviation can be though of as the risk of a certain percentage loss for a constant notional investor who keeps the same weight in the market over time. In contrast, the the tail risk relative to the conditional standard deviation can be though of as the probability of a certain loss for a volatility targeting investor who varies the position in the market over time to keep a constant portfolio volatility (and keeps the remainder in cash). Such a volatility targeting strategy is common practice among hedge funds and often used in academic research as well (Moskowitz, Ooi, and Pedersen (2012), Asness, Frazzini, and Pedersen (2012), Moreira and Muir (2017a), and Moreira and Muir (2017b)).

More formally, consider the unexpected return from an investment in the return on the market and the risk free asset,

$$r_{t,T}^{\text{i, shock}} = \omega_{t,T}^{i}(R_{t,T} - E_t[R_{t,T}])$$

where $\omega^i_{t,T}$ is the conditional portfolio weight on the market portfolio (and $1-\omega^i_{t,T}$ is the weight in the risk free asset). The conditional portfolio weight for the constant notional investors is constant. We assume that wieght is 1 for convenience. The portfolio weight of the volatility targeting investor is

$$\omega^{\text{vol target}}_{t,T} = \frac{\sigma^{\text{target}}}{\sigma_{t,T}}$$

where $\sigma_{t,T}$ is the ex-ante conditional volatility on the market. For convenience, we assume that the target volatility (σ^{target}) is the same as the time series average of the market portfolio, $\bar{\sigma}$. The probability of an unexpected loss which is lower than α for either investor is

$$\begin{aligned} P_t(r^{\text{i, shock}}_{t,T} < \alpha) &= P_t\left(\omega^{\text{i}}_{t,T}(R_{t,T} - E_t[R_{t,T}]) < \alpha\right) \\ &= P_t\left(R_{t,T} - E_t[R_{t,T}] < \frac{\alpha}{\omega^{\text{i}}_{t,T}}\right) \end{aligned}$$

If we, for instance, study the probability of realizing an unexpected loss that is more than two times the unconditional standard deviation of the market (i.e. $\alpha = 2\bar{\sigma}$), then the probabilities for the constant notional and volatility targeting investors are

$$P_t(r^{\text{constant, shock}}_{t,T} < -2\bar{\sigma})) = P_t\left(R_{t,T} - E_t[R_{t,T}] < -2\bar{\sigma}\right))$$

for the constant notional investor and

$$P_t(r^{\text{target, shock}}_{t,T} < -2\bar{\sigma}) = P_t\left(R_{t,T} - E_t[R_{t,T}] < -2\sigma_{t,T}\right)$$

for the volatility targeting investor. Accordingly, the probability is the same as the probability that the return to the market is two unconditional standard deviation below expectations; For the volatility targeting investor, the probability is the same as the probability that the return to the market is two conditional standard deviation below expectations. We thus interpret the results on the unconditional probabilities as the tail loss of a constant notional investor and the results on the conditional probabilities as the tail loss of the volatility targeting investor.

We next relate the tail probabilities to the conditional variance and past returns. Panel A of Table 5 reports correlations between variance and the probability of a portfolio return that is less than the threshold α. The first two columns of Panel A shows the correlations between log-variance and the probability that the market realizes an unexpected return less than $-2\bar{\sigma}$ and $-3\bar{\sigma}$. The correlations range from 0.80 to 0.94 with tight bootstrapped confidence bounds, showing that the conditional variance plays a large role in this probability.

More importantly, Panel A of Table 5 also reports correlations between log-variance and the probability of a portfolio return of $2\sigma_{t,T}$ and $3\sigma_{t,T}$, which reflect the portfolio risk of the volatility targeting investor. These probabilities are negatively correlated with log variance at -0.87 to -0.60, with tight bootstrapped confidence bounds. These high negative correlations show that the portfolio of the volatility targeting investor is exposed to substantially more risk at times when variance is low. This finding can help explain why Moreira and Muir (2017a) and Moreira and Muir (2017b) find that investors can earn high Sharpe ratios by moving wealth into the market at times of low variance and moving wealth out of the market when variance increases (in some sense mimicking a volatility targeting strategy). The relatively (to variance) high expected return in calm periods may be compensation for the elevated higher-moment risks.

We next investigate the relation between tail probabilities and past returns. Specifically, we regress tail probabilities onto past two year returns, e.g., the probability of a $-2\sigma_{t,T}$ drop as

$$P_t(r_{t,T}^{\text{shock}} < -2\sigma_{t,T}) = \beta_0 + \beta_1 r_{t-24,t} + \epsilon_{t,T} \tag{16}$$

Panel B of Table 5 reports β_1 coefficients from regressions such as in (16). We find that the probability of both a $-2\sigma_{t,T}$ and a $-3\sigma_{t,T}$ drop in the market is statistically significant and positively related to past returns. The economic magnitude is such that a 50% market run-up over the past two years implies a 1% higher probability of a monthly $-2\sigma_{t,T}$ drop in the market. Furthermore, the monthly probability of a $-2\bar{\sigma}$ drop in the market is negatively related to past returns, which is to be expected, because this probability is highly correlated to variance, as shown in Table 4, and periods after market run-ups are usually associated with low variance. Panel C of Table 5 reports β_1 coefficients from regressions such as in (16) when controlling for lagged probabilities on the right hand side. Controlling for lagged probabilities does not change our results: high past two year returns imply higher current tail probabilities for the volatility targeting investor.

7 Economic Implications: Higher-Moment Risk, Uncertainty, and Business Cycles

Time variation in higher-moment risk may also have important implications for the real economy. Indeed, the growing literature on uncertainty shocks argues that uncertainty about the economy may drive drive business cycles because higher uncertainty deters firms from investing. This may happen through multiple mechanisms, but often the option value of postponing investments is stressed: higher uncertainty about a project increases the option value of investment projects, and managers may thus prefer to hold on to that option rather than undertaking the investment (Dixit and Pendyck, 1993). Similarly, more higher-moment risk increases the option value of an investment project and therefore increases the incentive to postpone investment (Dixit and Pindyck, 1993).[4] We therefore test whether more higher-moment risk predicts lower future industrial production.

At each time t, we regress the change in industrial production between period t and $t+i$ onto the ex ante probability of a two-sigma loss:

$$\mathrm{IND}_{t+i} - \mathrm{IND}_t = \beta_0 + \beta_1 P_t(r_{t,T}^{\text{shock}} < -2\sigma_{t,T}) + \epsilon_{t,T}$$

The results are presented in Panel A of Table 6. For the one-year change in industrial production, the loading on the tail risk is insignificant, suggesting that tail risk does not reduce investment on the one-year horizon. However, on the two and three year horizon, the estimate is negative and statistically significant when measuring tail risk at the quarterly horizon. Accordingly, a high tail risk predicts lower future industrial production. We note that the lack of predictability on the one year horizon could arise from the fact that it takes time for firms to adjust their investments, and it takes additional time for their adjustments to ripple through the economy. Finally, we also find that policital uncertainty does not predict changes in industrial production.

In the above regressions, one may be concerned that causality runs the opposite way, and that it is recessions that cause uncertainty and not the other way around

[4] Merton (1976) shows that, keeping the first moment constant, higher tail risk increases the value of a call option.

(Berger, Dew-Becker, and Giglio, 2017). For instance, in the financial crisis volatility spiked in October 2008, which was probably after the great recession had set in motion, meaning that the uncertainty was unlike to have started the recession. To mitigate such concerns, we next run similar regressions where we skip the first year of industrial production, meaning we forecast the change in production from one year ahead and onwards:

$$\text{IND}_{t+i} - \text{IND}_{t+12} = \beta_0 + \beta_1 P_t(r_{t,T}^{\text{shock}} < -2\sigma_{t,T}) + \epsilon_{t,T}$$

These results from regression (7) are presented in Panel B of Table 6. Tail risk still predicts changes in industrial production negatively at both the two and three year horizon. The results are substantially stronger than in Panel A, with R^2 as high as 22%, suggesting that tail risk may play an important role in business cycle fluctuations. In constrast, political uncertainty predicts future indstrial production positively, not negatively, questioning its importance in understanding business cycle fluctuations. The strong relation between tail risk and industrial production is depcited in Figure 10.

Finally, as a word of caution, the results of the predictive regressions in Table 6 are based on a small sample and the inference might thus be subject to small sample issues. For instance, the t-statistics based on Newey West are going to be biased upwards. In untabulated results, we redo the analysis using standard errors of Lazarus, Lewis, and Stock (2017), that are robust to small samples, and find that our results remain significant.

8 What Explains Time Variation in Higher-Moment Risk?

In this final section, we next study why higher-moment risk varies over time, and in particular why it is high when volatility is low and past returns are high. We consider two potential explanations. The first is that low volatility spurs financial sector leverage and thereby creates endogenous higher-moment risk as in Brunnermeier and Sannikov (2014b). The second potential explanation is that high past returns makes investors worried that prices are unstainable which is reflected in a higher perceived probability of a crash (tail loss).

8.1 Higher-Moment Risk and the Volatility Paradox

The time variation we uncover in higher-moment risk is conceptually consistent with the model by Brunnermeier and Sannikov (2014a). Brunnermeier and Sannikov model a macro economy with a financial sector and document a volatility paradox, which is the notion that endogenous risk is high even though exogenous risk is low. One can think of exogenous risk as volatility and endogenous risk as higher-moment risk, meaning that their result is consistent with our findings: higher-moment risk is high when volatility is low.

In their model, the the volatility paradox arises because times with low volatility are times where the financial sector is the most levered and thus most sensitive to a negative funding liquidity shock that will cause sophisticated investors to delever and potentially induce a firesale. According to this, we should expect higher moment risk to be high when sophisticaed investors are more levered. We therefore regress measures of higher moment risk onto the ex ante level of financial intermediary leverage as measured by He, Kelly, and Manela (2016):

$$\mathrm{HMR}_{t,T} = \beta_0 + \beta_1 \mathrm{Leverage}_t + \epsilon_{t,T} \tag{17}$$

where the risk, $\mathrm{HMR}_{t,T}$, is one of the following measures of higher moment risk: the risk of a tail loss, the higher-moment risk index (HRI), skewness, kurtosis, hyperkurtosis, or hyperskewness. We also consider variance as a left hand side variable.

Panel A of Table 7 shows the results of regression (17). We do not find a relation between ex ante intermediary leverage and any of our measures of higher-moment risk. However, we do find a positive relation between intermediary leverage and variance. These results are inconsistent with our hypothesis, but they are likely driven by the fact that the measure of intermediary leverage in He, Kelly, and Manela (2016) is counter-cyclical, which is also inconsistent with our hypothesis. Intuitively, one may expect intermediary leverage to be pro-cyclical and low during bad times because investors unwind some of their levered positions, but their equity value also falls meaning that leverage may increase.

We instead consider the relation between funding liquidity and higher moment risk. Higher funding liquidity may cause investors to lever up more and thereby contribute to the volatility paradaox, meaning we should expected higher liquidity to generate more higher-moment risk. In addition, market and funding liquidity are interrelated (Brunnermeier and Pedersen, 2009), so we consider both market and funding liquidity as independent variables in regression (17). We measure market liquidity by the average bid-ask spread in the S&P 500 and we measure funding liquidity as the TED spread. The results are presented in Table 8. Consistent with our expectations, higher-moment risk is negatively related to both the bid-ask spread and the ted spread, meaning that higher-moment risk is lower in more illiquid markets (and higher in more liquid markets). The relations between the higher-moment risk index and funding and market liqudity are depicted in Figure 11 and 12.

8.2 "Bubble" Characteristics

Given the positive relation between past returns and higher-moment risk, one may hypothesize that higher-moment risk is a symptom of bubbles: following run ups, investors worry that current prices represent a bubble that may burst, which is reflected in a higher probability of a crash. To test this hypothesis, we regress the higher-moment risk index on a range variables that have been proposed as possible indicators of bubbles:

$$\mathrm{HRI}_{t,T} = \beta_0 + \beta_1 \mathrm{Characteristic}_t + \epsilon_{t,T} \tag{18}$$

The characteristics we consider are: high price acceleration (Greenwood, Shleifer, and You, 2017), high turnover (Chen, Hong, and Stein, 2001), and high equity issuance (Pontiff and Woodgate, 2008). Price acceleration is defined as the annualized past two year return minus the return of the first of the two years. Acceleration captures the convexity in the recent price path, and a high acceleration is intended to be associated bubbles. Issuance is defined as the percentage of firms in the S&P 500 index that issued equity in the past year. We follow Greenwood, Shleifer, and You (2017), and define an equity issuance as the event that a firm's split-adjusted share count increased by five percent or more. We also consider valuation measures. Shiller (1996) argue that these measures capture irrational expectations, but they may reflect rationally expected discount rates and are thus not unique to the concept of bubbles. In addition, the use of valuation measures is problematic because they themselves are products of the amount of risk investors face (including higher-moment risks). Regardless, we include the valuation measures for completeness.

Table 9 reports the results of regression (18). We find no reliable evidence that the bubble characteristics predict higher-moment risk. The only bubble characteristics that is statistically significantly related to higher-moment risk on its own is turnover, but the sign is wrong: the regression predicts that higher turnover leads to lower higher-moment risk. When interacting turnover and issuance with past returns, we get the right sign, and statistical significance, for turnover, but the wrong sign, and statistical significance, for issuance. The cay variable from Lettau and Ludvigson (2001) is, however, negatively associated with higher-moment risk, suggesting that higher-moment risk is high when wealth is high realtive to consumption.

9 Conclusion

We employ a new method to estimate forward looking higher-order moments of the return distribution for the S&P 500 from option prices. Using this method, we document new stylized facts about higher-moment risk: (1) Higher-moment risk comove strongly in the sense that the third to sixth moment tend to become riskier at the same time. (2) The comovement is so strong that 91% of their comovement is explained by their first principal componenet. (3) Higher-moment risk is high when the second moment, variance, is low. (4) Higher-moment risk is high when past returns are high.

These new stylized facts have important implications for financial markets and investors. The variation in higher moment risk causes the risk of a two-sigma event to vary from 3.3% to 11% over the sample. This probability is higher in periods of low variance, creating additional risk for investors who lever their portofolios when variance is low, such as volatility targeting investors, who should thus take this additional higher-moment risk into account when creating portfolios. In addition, financial regulators should also worry that volatility targeting invetors, such as hedge funds, load up on tail risk in calm periods, thereby exposing themselves and the financial system to more risk. These results echo the findings of Brunnermeier and Sannikov (2014), who shows theoretically that the economy and the financial system becomes more vulnerable when volatility is low because sophisticated investors apply more leverage and creates endogenous risk.

Finally, higher-moment risk is also related to future industrial output. When higher-moment risk is high, firms produce less during the subsequent three years. The correlation between future industrial output and higher-moment risk is as high as

-0.47, suggesting a potentially important role for higher-moment risk in understanding firm investments. In addition, higher-moment risk predicts industrial production better than political uncertainty, suggesting that higher-order moments of returns are more relevant measures for understanding the impact of uncertainty and uncertainty shocks on the business cycle.

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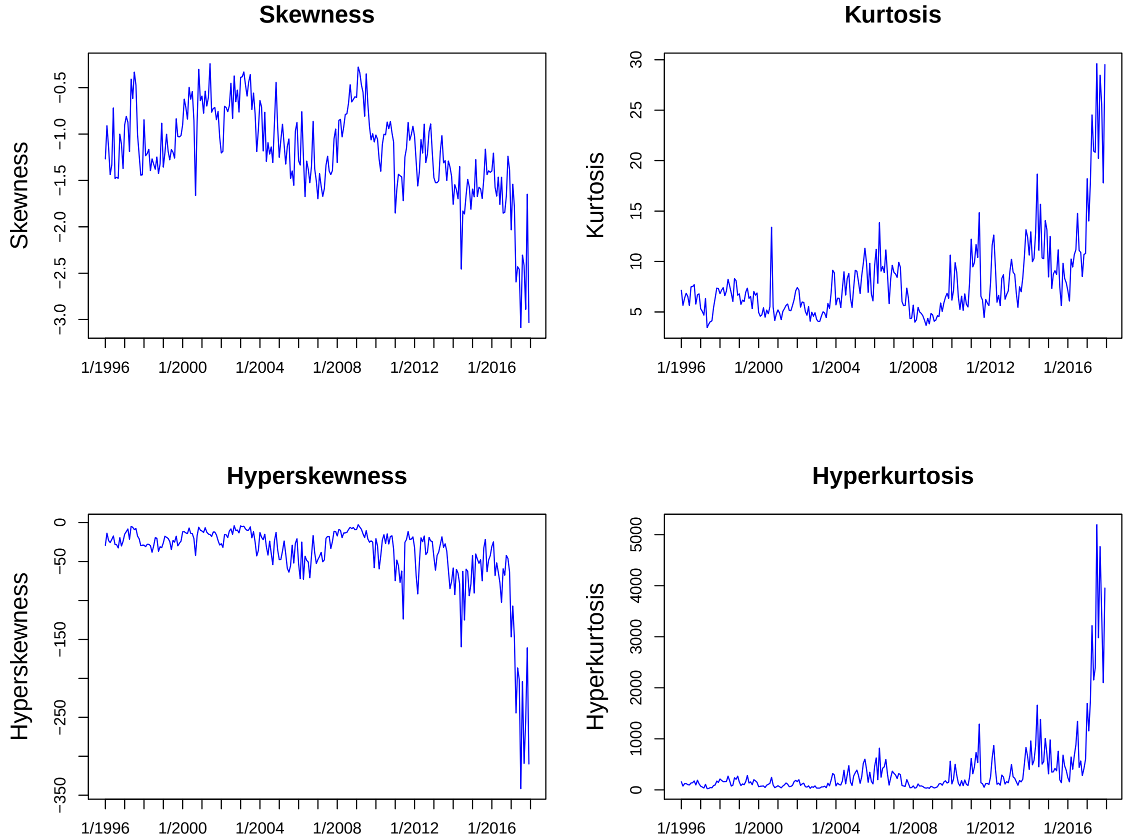


Figure 1: **Higher Order Moments of the Return Distribution of the S&P 500 (Monthly Horizon).** The figures show a time series plot of monthly higher order moments for the S&P 500 index.

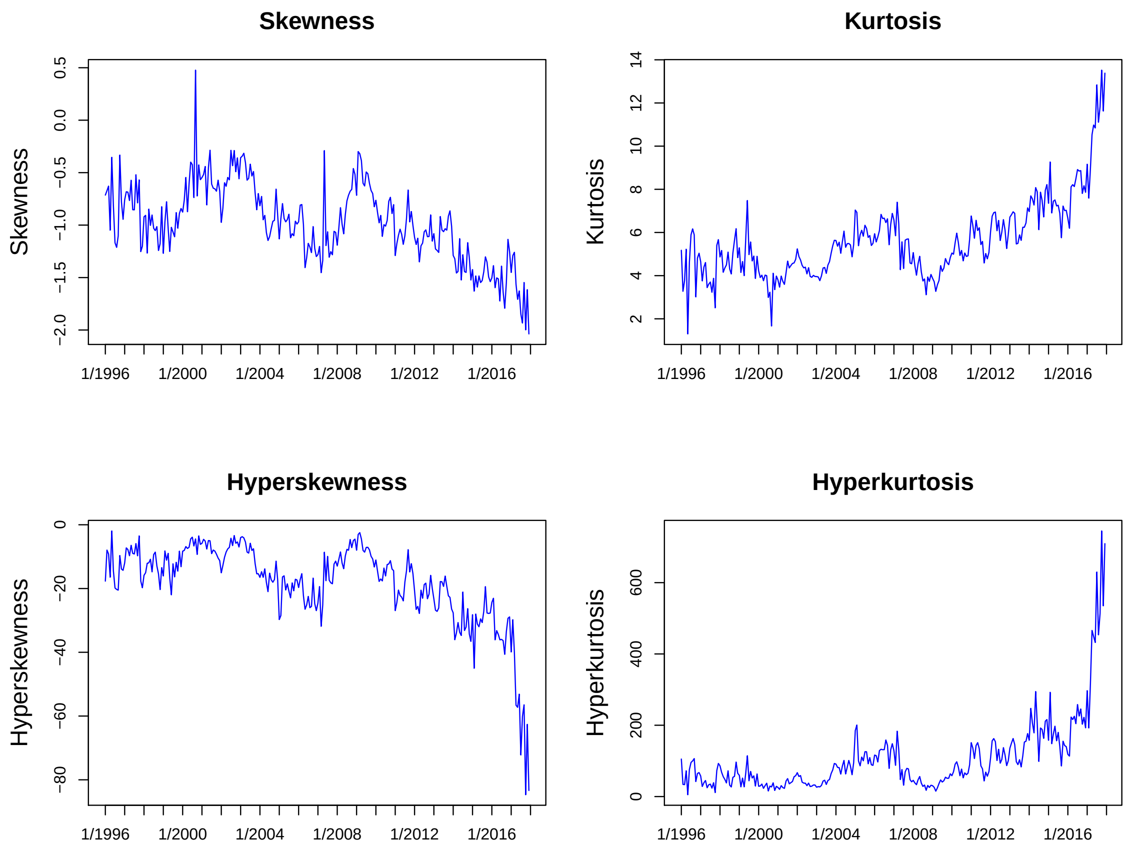
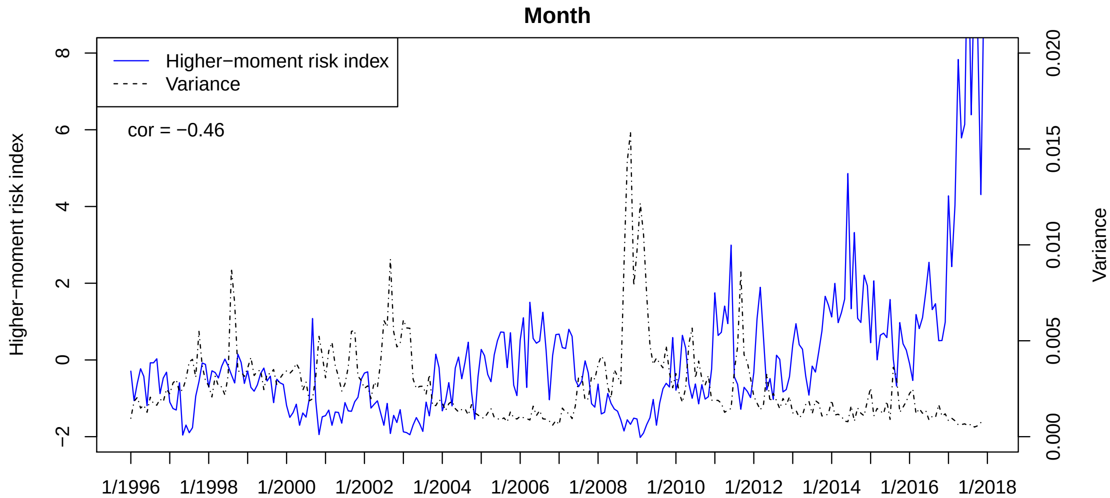


Figure 2: **Higher Order Moments of the Return Distribution of the S&P 500 (Quarterly Horizon).** The figures show a time series plot of quarterly higher order moments for the S&P 500 index.



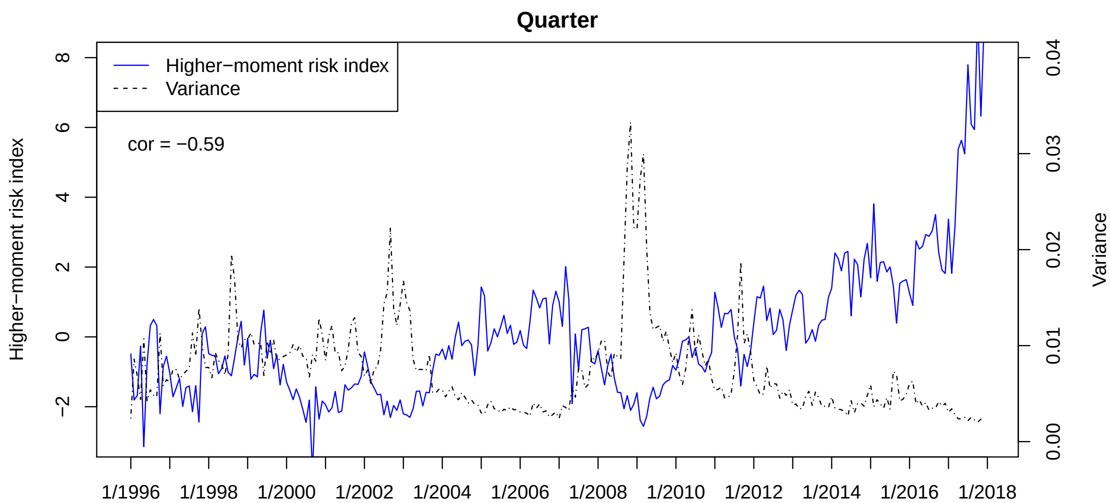


Figure 3: **The Higher-Moment Risk Index and Variance.** This figure shows time series plots of the monthly and quarterly higher-moment risk index and the second moment (variance) of the return distribution. The higher-moment risk index is the first principal component of the space spanned by skewness, kurtosis, hyperskewness, and hyperkurtosis. The higher the index the more higher-moment risk.

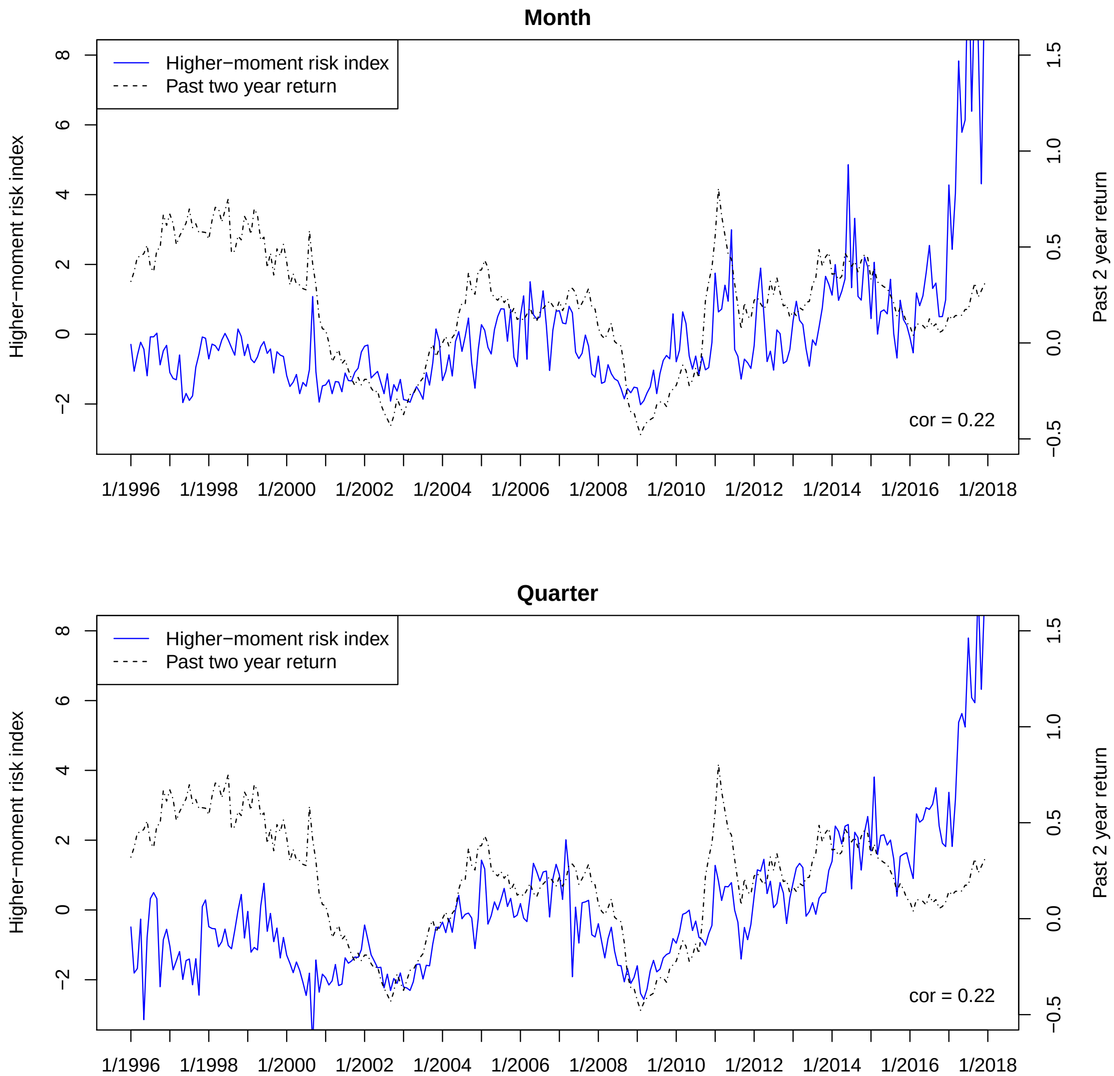


Figure 4: **The Higher-Moment Risk Index and the Past Two Year Returns.** This figure shows time series plots of the monthly and quarterly higher-moment risk index and the past two-year return on the S&P 500. The higher-moment risk index is the first principal component of the space spanned by skewness, kurtosis, hyper-skewness, and hyperkurtosis. The higher the index the more higher-moment risk.

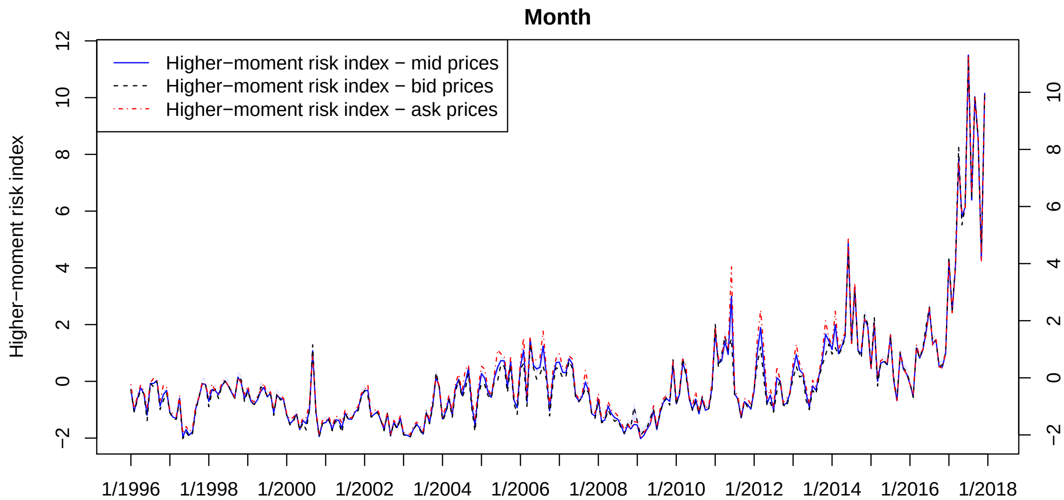


Figure 5: **The Higher-Moment Risk Index Using Bid or Ask Prices.** This figure shows the monthly horizon higher-moment risk indexes when using either bid, ask, or mid prices. The pairwise correlation between the time series range from 0.988 to 0.998, suggesting that our choice of mid prices does not affect our results.

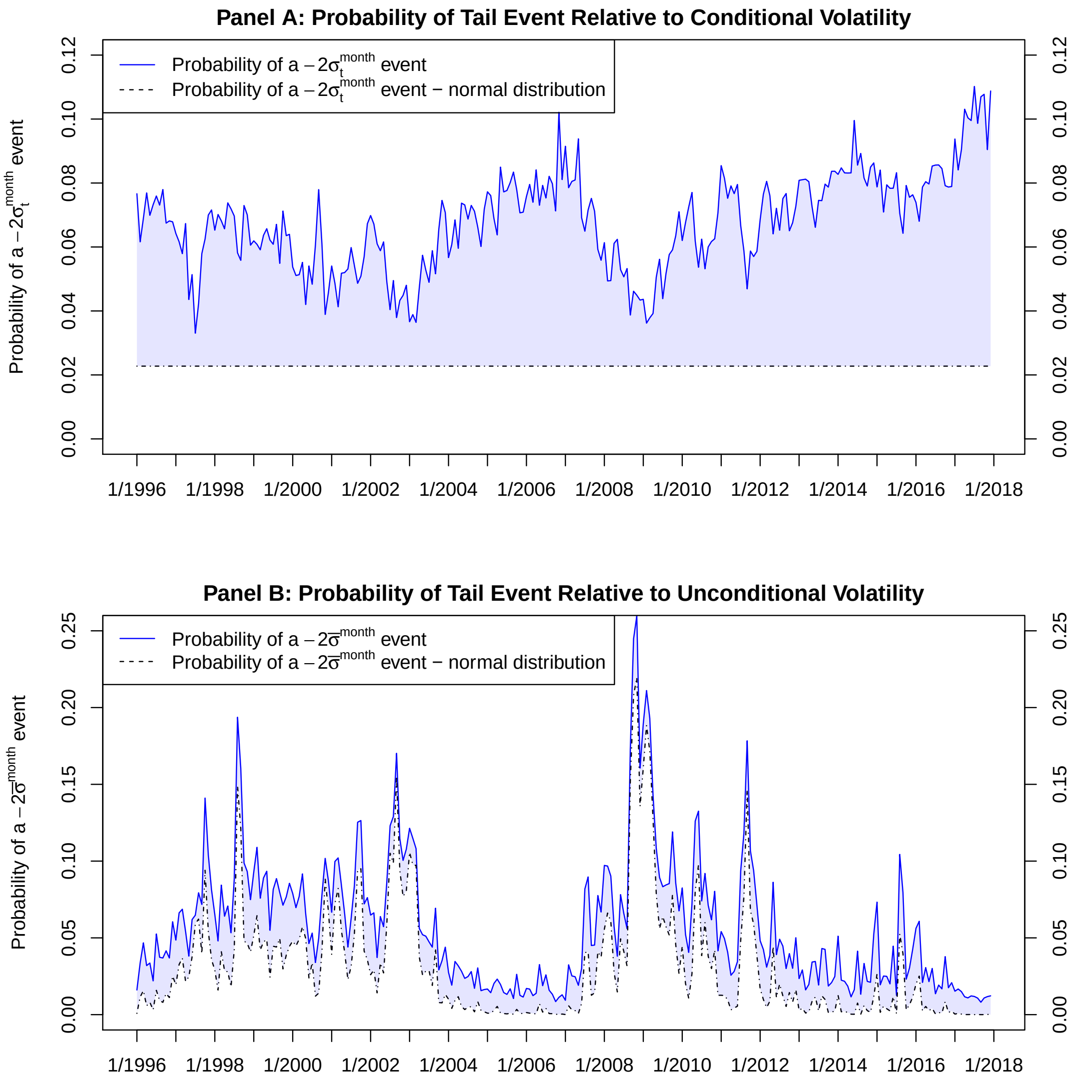


Figure 6: **Probabilities of a Two-Sigma Loss on the S&P 500 (Monthly Horizon).** Panel A shows the probability that the monthly S&P 500 return is two conditional standard deviations ($2\sigma_t^{\text{month}}$) below its its expected value. The solid blue line shows the probability as derived from option prices and the dotted line shows the probability implied by the normal distribution, which is a constant 2.5%. The shaded area between the lines represents higher-moment risk. Panel B shows the probability that the monthly S&P 500 return is two unconditional standard deviations below its expected value. The solid blue line shows the probability as derived from option prices and the dotted line shows the probability implied by the normal distribution. The unconditional standard deviation is the time series average of 4.9% per month. The shaded area between the lines represents higher-moment risk.

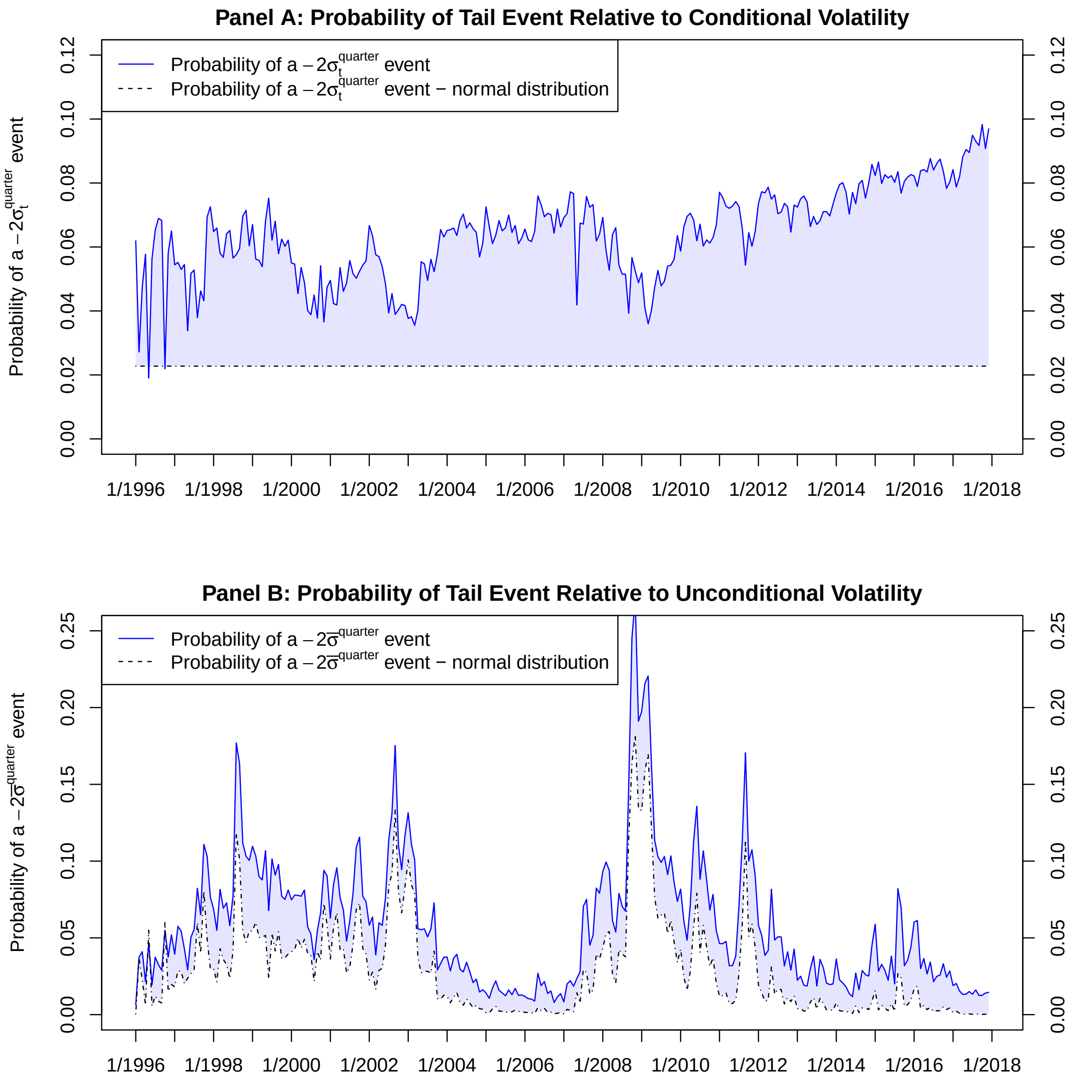


Figure 7: **Probabilities of a Two-Sigma Loss on the S&P 500 (Quarterly Horizon).** Panel A shows the probability that the quarterly S&P 500 return is two conditional standard deviations ($2\sigma_t^{\text{month}}$) below its its expected value. The solid blue line shows the probability as derived from option prices and the dotted line shows the probability implied by the normal distribution, which is a constant 2.5%. The shaded area between the lines represents higher-moment risk. Panel B shows the probability that the quarterly S&P 500 return is two unconditional standard deviations below its expected value. The solid blue line shows the probability as derived from option prices and the dotted line shows the probability implied by the normal distribution. The unconditional standard deviation is the time series average. The shaded area between the lines represents higher-moment risk.

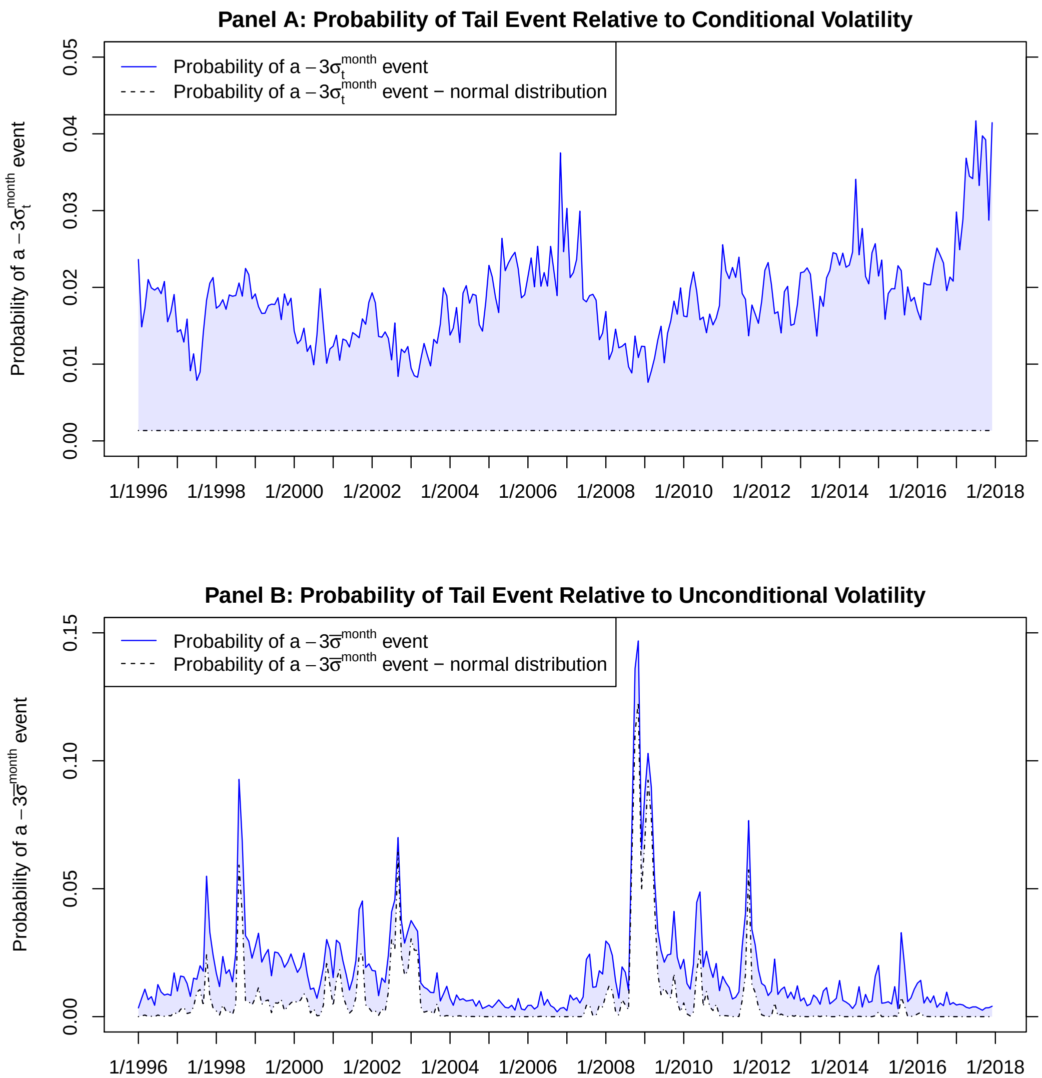


Figure 8: **Probabilities of a Three-Sigma Loss on the S&P 500 (Monthly Horizon).** Panel A shows the probability that the monthly S&P 500 return is three conditional standard deviations ($3\sigma_t^{\text{month}}$) below its its expected value. The solid blue line shows the probability as derived from option prices and the dotted line shows the probability implied by the normal distribution, which is a constant 0.13%. The shaded area between the lines represents higher-moment risk. Panel B shows the probability that the monthly S&P 500 return is three unconditional standard deviations below its expected value. The solid blue line shows the probability as derived from option prices and the dotted line shows the probability implied by the normal distribution. The unconditional standard deviation is the time series average of 4.9% per month. The shaded area between the lines represents higher-moment risk.

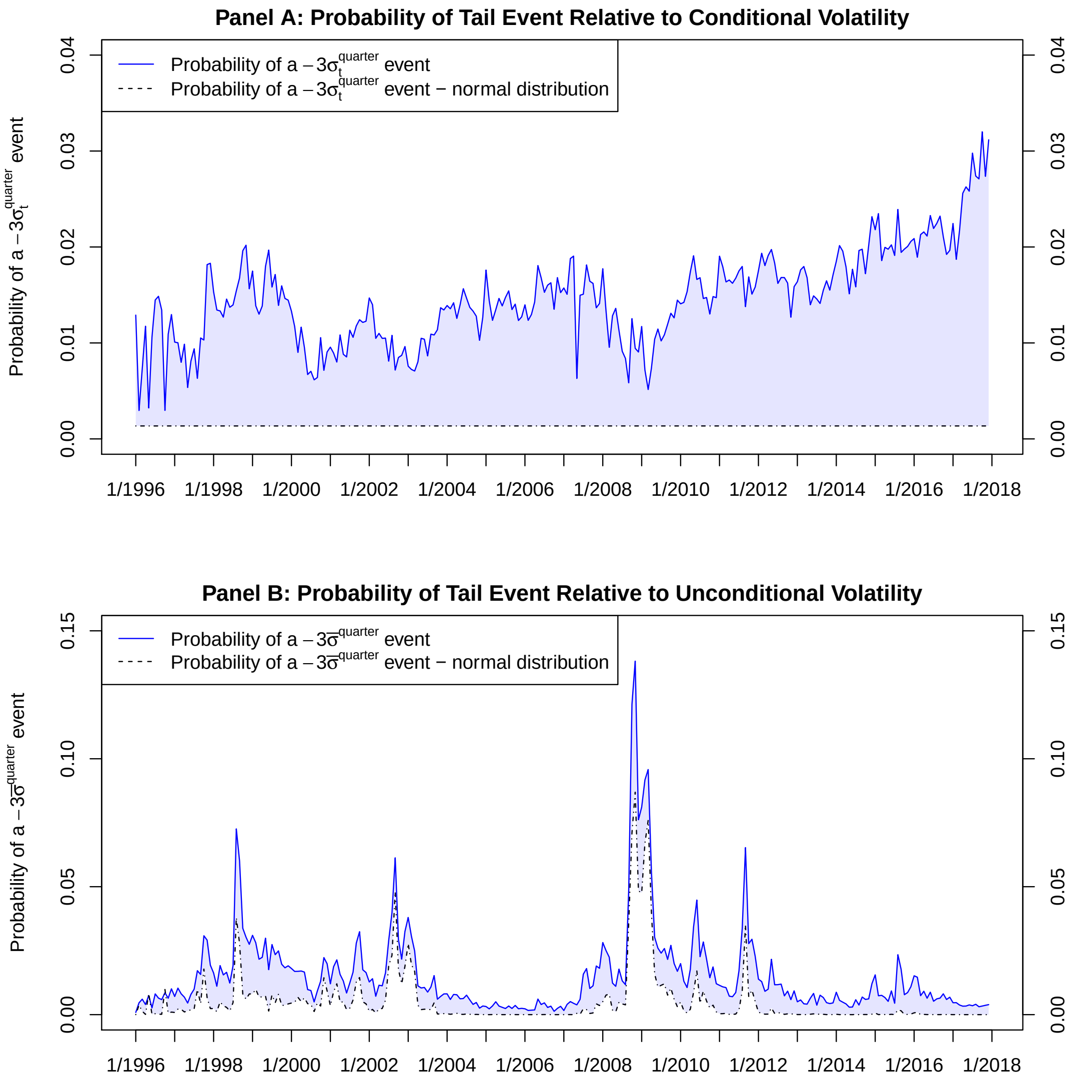


Figure 9: **Probabilities of a Three-Sigma Loss on the S&P 500 (Quarterly Horizon).** Panel A shows the probability that the quarterly S&P 500 return is three conditional standard deviations ($3\sigma_t^{\text{month}}$) below its its expected value. The solid blue line shows the probability as derived from option prices and the dotted line shows the probability implied by the normal distribution, which is a constant 2.5%. The shaded area between the lines represents higher-moment risk. Panel B shows the probability that the quarterly S&P 500 return is three unconditional standard deviations below its expected value. The solid blue line shows the probability as derived from option prices and the dotted line shows the probability implied by the normal distribution. The unconditional standard deviation is the time series average. The shaded area between the lines represents higher-moment risk.

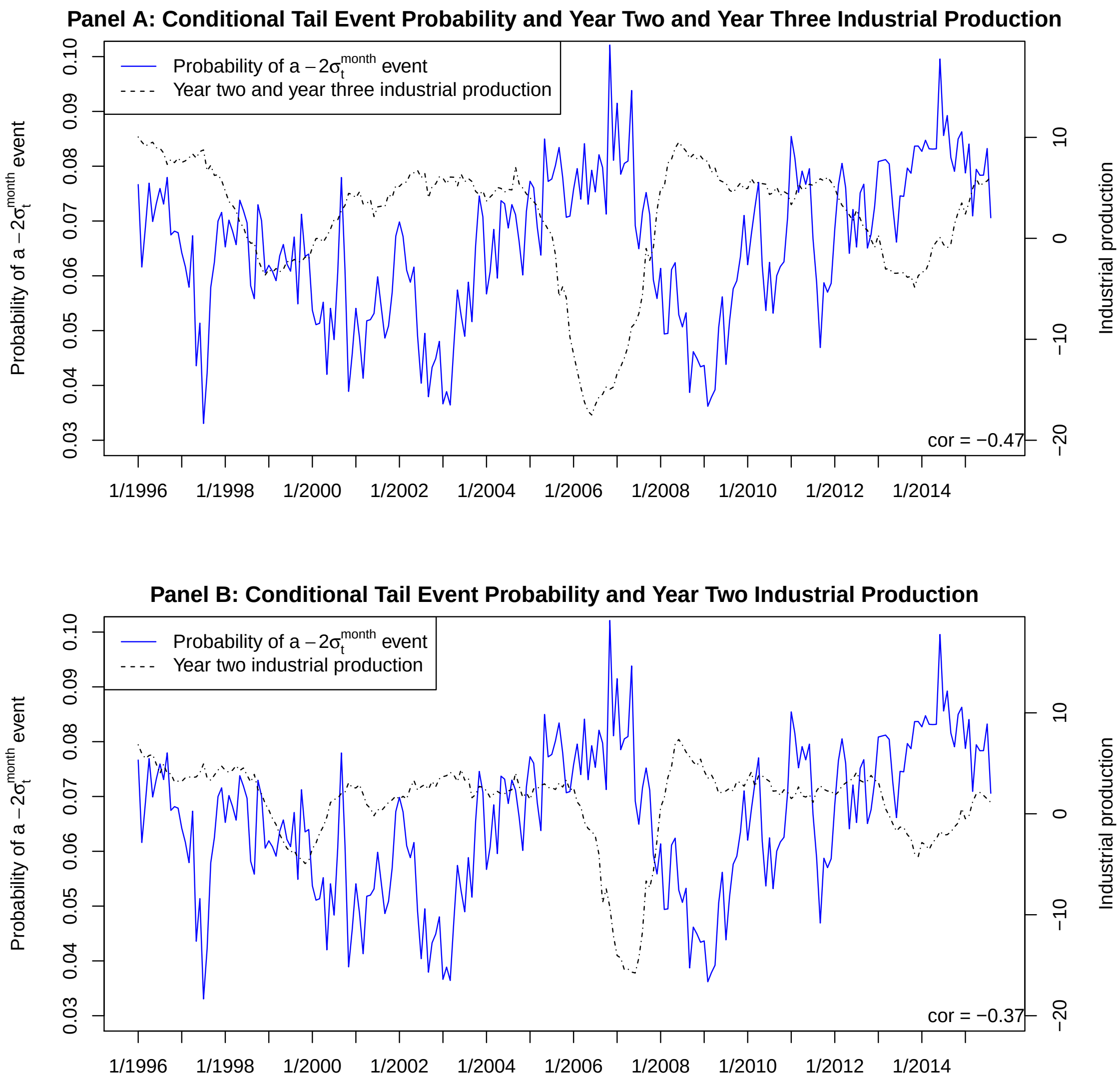


Figure 10: **Conditional Tail Loss Probabilities and Industrial Production.** Panel A shows time series of the probability of a $-2\sigma_t^{\text{month}}$ event (solid) along with the level of industrial production three years ahead minus the level of industrial production one year ahead (dashed). Panel B shows time series of the probability of a $-2\sigma_t^{\text{month}}$ event (solid) along with the level of industrial production two years ahead minus the level of industrial production one year ahead (dashed).

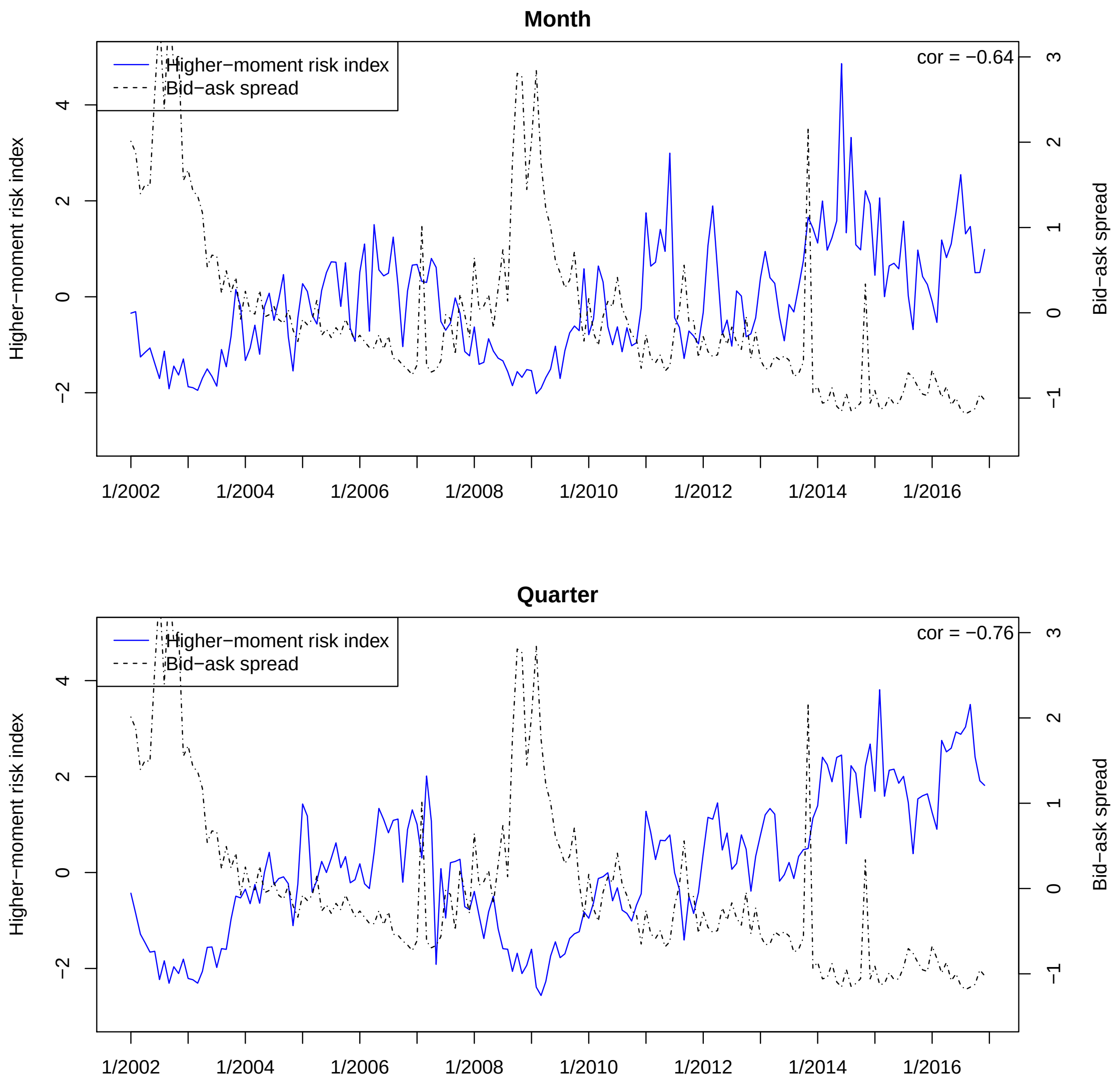


Figure 11: **Higher-Moment Risk Index and Market Illiquidity.** This figure shows time series plots of the higher-moment risk index and market illiquidity, which is measured as the average value-weighted bid-ask spread of S&P 500 constituents.

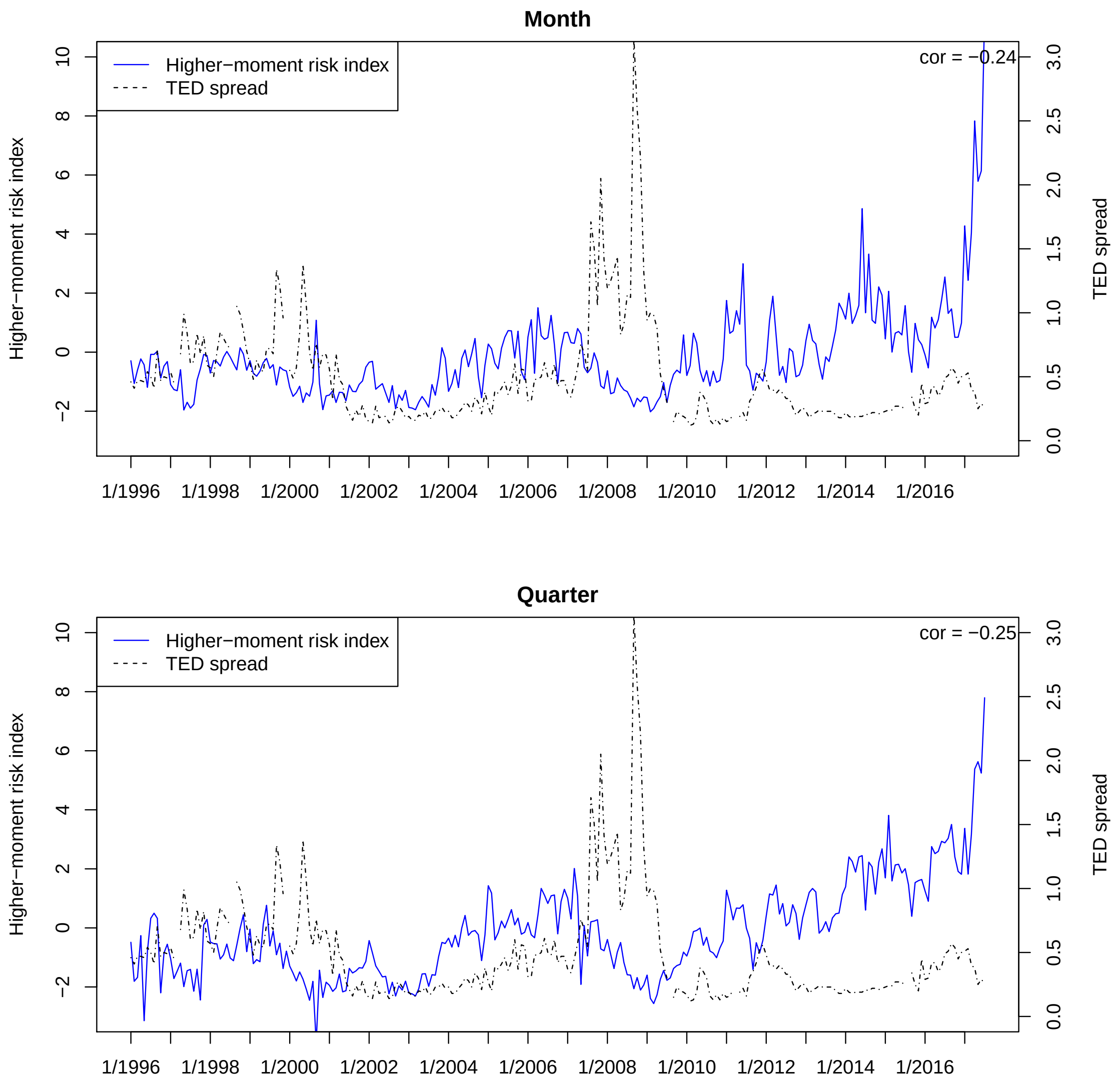


Figure 12: **Higher-Moment Risk Index and Funding Illiquidity.** This figure shows time series plots of the higher-moment risk index and funding illiquidity, which is measured as the TED spread.

Table 1: **Summary Statistics of S&P 500 Moments.** This table reports the average time-series values of the following ex ante moments of the return distribution of the $S\&P500$: excess return (ER−Rf), standard deviation (St. dev.), skewness (Skew), kurtosis (Kurt), hyperskewness (Hskew), and hyperkurtosis (Hkurt). We estimate ex ante moments from the point of view of a risk-neutral investor ($\gamma = 0$), a log-utility investor ($\gamma = 1$), and two power-utility investors ($\gamma = 3$, $\gamma = 5$).

		Annualized (%)					
Horizon	Risk-aversion	ER−Rf	St. dev.	Skew	Kurt	Hskew	Hkurt
Month	$\gamma = 0$	0	19.09	-1.57	10.38	-65.23	591.79
Month	$\gamma = 1$	4.15	18.20	-1.42	9.47	-56.47	517.61
Month	$\gamma = 3$	11.26	16.89	-1.16	8.02	-40.87	368.92
Month	$\gamma = 5$	17.34	15.99	-0.95	7.02	-29.35	257.60
Quarter	$\gamma = 0$	0	19.73	-1.27	6.59	-27.01	164.74
Quarter	$\gamma = 1$	4.25	18.40	-1.17	6.21	-23.56	140.20
Quarter	$\gamma = 3$	11.05	16.54	-0.98	5.63	-18.29	105.91
Quarter	$\gamma = 5$	16.48	15.32	-0.80	5.17	-14.00	81.72

Table 2: **Correlations Between S&P 500 Moments.** This table reports the pairwise correlations between the following S&P 500 moments: expected return (Er), variance (Var), skewness (Skew), kurtosis (Kurt), hyperskewness (Hskew), and hyperkurtosis (Hkurt). Panel A reports results for monthly moments and Panel B reports results for the quarterly moments. We report 95% bootstrapped confidence bounds in brackets.

Panel A: Month

	Er	Var	−Skew	Kurt	−Hskew	Hkurt
Er	1	0.99	−0.48	−0.46	−0.39	−0.32
		[0.99,1]	[−0.56,−0.41]	[−0.52,−0.41]	[−0.46,−0.35]	[−0.39,−0.28]
Var		1	−0.53	−0.49	−0.42	−0.34
			[−0.61,−0.47]	[−0.55,−0.45]	[−0.49,−0.38]	[−0.41,−0.31]
−Skew			1	0.86	0.81	0.73
				[0.81,0.89]	[0.76,0.86]	[0.65,0.79]
Kurt				1	0.98	0.93
					[0.97,0.99]	[0.92,0.96]
−Hskew					1	0.98
						[0.98,0.99]
Hkurt						1

Panel B: Quarter

	Er	Var	−Skew	Kurt	−Hskew	Hkurt
Er	1	0.99	−0.50	−0.53	−0.53	−0.47
		[0.99,0.99]	[−0.58,−0.43]	[−0.59,−0.49]	[−0.60,−0.50]	[−0.53,−0.43]
Var		1	−0.58	−0.59	−0.59	−0.52
			[−0.65,−0.52]	[−0.65,−0.56]	[−0.65,−0.55]	[−0.58,−0.47]
−Skew			1	0.85	0.85	0.73
				[0.82,0.87]	[0.82,0.88]	[0.69,0.77]
Kurt				1	0.98	0.95
					[0.97,0.99]	[0.93,0.96]
−Hskew					1	0.97
						[0.97,0.98]
Hkurt						1

Table 3: **Principal Components of Higher-Moment Risks.** This table presents the results of a principal component analysis of the ex ante skewness (Skew), kurtosis (Kurt), hyperskewness (Hskew), and hyperkurtosis (Hkurt) of the S&P 500. Panel A reports results on for the monthly moments and Panel B reports results for the quarterly moments. The rightmost column shows the amount of variation explained by the various principal components. The last row in each panel shows the correlation between the first principal component and the different moments.

Panel A: Monthly horizon

	Skew	Kurt	Hskew	Hkurt	Variation explained
PC 1 eigenvector	−0.46	0.52	−0.52	0.50	91%
PC 2 eigenvector	−0.84	−0.04	0.24	−0.48	8%
PC 3 eigenvector	−0.28	−0.80	−0.03	0.53	1%
PC 4 eigenvector	0.06	−0.30	−0.82	−0.48	0%
PC 1 correlation	−0.88	0.99	−0.99	0.96	

Panel B: Quarterly horizon

	Skew	Kurt	Hskew	Hkurt	Variation explained
PC 1 eigenvector	−0.47	0.52	−0.52	0.50	92%
PC 2 eigenvector	−0.84	−0.10	0.15	−0.52	7%
PC 3 eigenvector	−0.21	−0.84	−0.23	0.43	1%
PC 4 eigenvector	−0.20	0.11	0.81	0.54	0%
PC 1 correlation	−0.89	0.99	−0.99	0.96	

Table 4: **Cyclicality in Higher-Moment Risks.** Panel A reports correlations between ex ante variance and the higher-moment risk index (HRI). Panel B reports correlations between the past two year returns of the S&P 500 index and the higher-moment risk index (HRI). Panel C reports correlations between the past two-year returns of the S&P 500 index and the individual higher order moments: variance (Var), skewness (Skew), kurtosis (Kurt), hyperskewness (Hskew), and hyperkurtosis (Hkurt). We report bootstrapped 95% confidence intervals in brackets.

Panel A: Variance and the higher-moment risk index

Horizon	HRI
Month	−0.46
95% CI	[−0.53,−0.42]
Quarter	−0.59
95% CI	[−0.65,−0.56]

Panel B: Past return and the higher-moment risk index

Horizon	HRI
Month	0.22
95% CI	[0.14, 0.31]
Quarter	0.22
95% CI	[0.13, 0.32]

Panel C: Past return and individual higher order moments

Horizon	Var (%)	Skew	Kurt	Hskew	Hkurt
Month	−0.43	−0.38	0.20	−0.16	0.11
95% CI	[−0.52, −0.31]	[−0.48, −0.28]	[0.12, 0.29]	[−0.24, −0.10]	[0.05, 0.19]
Quarter	−0.42	−0.34	0.15	−0.22	0.15
95% CI	[−0.52, −0.30]	[−0.45, −0.23]	[0.06, 0.26]	[−0.31, −0.13]	[0.07, 0.24]

Table 5: **Tail Risk, Variance, and Past Returns.** Panel A reports correlations between ex ante log-variance and tail loss probabilities. The probabilities are $P(r_t^h < -2\bar{\sigma}^h)$ and $P(r_{t,T} < -2\sigma_{t,T})$ where $r_{t,T} = R_{t,T} - E_t[R_{t,T}]$, $\sigma_{t,T}$ is the ex ante volatility from time t to T, and we define $\bar{\sigma}$ as the time series average of $\sigma_{t,T}$ at the appropriate horizon. We report bootstrapped 95% confidence intervals in brackets. Panel B reports regression slope coefficients when regressing physical tail loss probabilities onto the past two year returns. Panel C reports coefficients when controlling for lagged tail loss probabilities on the right hand side. We report standard errors in parentheses and significance as: * when $p < 0.1$, ** when $p < 0.05$, and *** when $p < 0.01$. We correct standard errors for autocorrelation using Newey and West (1987).

Panel A: Correlations between log-variance and tail probabilities

Horizon	$P_t(r_{t,T} < -2\bar{\sigma}$	$P_t(r_{t,T} < -3\bar{\sigma})$	$P_t(r_{t,T} < -2\sigma_{t,T})$	$P_t(r_{t,T} < -3\sigma_{t,T})$
Month	0.94	0.83	−0.87	−0.76
95% CI	[0.93,0.95]	[0.80,0.86]	[−0.89, −0.84]	[−0.80,−0.71]
Quarter	0.93	0.80	−0.71	−0.60
95% CI	[0.92,0.95]	[0.77,0.84]	[−0.77,−0.66]	[−0.67,−0.52]

Panel B: Tail probabilities (%) and past return

Horizon	$P_t(r_{t,T} < -2\bar{\sigma}$	$P_t(r_{t,T} < -3\bar{\sigma})$	$P_t(r_{t,T} < -2\sigma_{t,T})$	$P_t(r_{t,T} < -3\sigma_{t,T})$
Month	−5.52	−2.48	2.07**	0.68***
(s.e.)	(3.51)	(1.64)	(0.81)	(0.25)
Quarter	−5.97	−2.33	1.24	0.38
(s.e.)	(4.19)	(1.69)	(0.79)	(0.25)

Panel C: Tail probabilities (%) and past return - controlling for lagged probabilities

Horizon	$P_t(r_{t,T} < -2\bar{\sigma}$	$P_t(r_{t,T} < -3\bar{\sigma})$	$P_t(r_{t,T} < -2\sigma_{t,T})$	$P_t(r_{t,T} < -3\sigma_{t,T})$
Month	−1.09*	−0.60*	0.41**	0.16**
(s.e.)	(0.62)	(0.32)	(0.18)	(0.08)
Quarter	−0.92	−0.47*	0.28*	0.06
(s.e.)	(0.57)	(0.28)	(0.15)	(0.04)

Table 6: **Tail Risk and Industrial Production.** This table reports the results of rolling monthly predictive regression of industrial production (IND) on the conditional probability of a tail event. We report standard errors in parentheses and significance as: * when $p < 0.1$, ** when $p < 0.05$, and *** when $p < 0.01$. We correct standard errors for autocorrelation using Newey and West (1987).

Panel A: Tail risk and changes in industrial production

	Dependent variable:								
	$IND_{t+12} - IND_t$			$IND_{t+24} - IND_t$			$IND_{t+36} - IND_t$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$P_t(r_{t,t+1} < -2\sigma_{t,t+1})$	0.59			-0.29			-0.58^{**}		
(s.e.)	(0.42)			(0.30)			(0.23)		
$P_t(r_{t,t+3} < -2\sigma_{t,t+3})$		0.03			-0.40^{*}			-0.55^{***}	
(s.e.)		(0.44)			(0.22)			(0.16)	
Economic policy uncertainty			-2.69			10.60			13.13
(s.e.)			(9.03)			(8.76)			(10.10)
No. obs.	260	260	260	248	248	248	236	236	236
Adj. R^2	0.02	-0.00	-0.00	0.01	0.03	0.03	0.09	0.09	0.06

Panel B: Tail risk and changes in industrial production – skipping one year

	Dependent variable:					
	$IND_{t+24} - IND_{t+12}$			$IND_{t+36} - IND_{t+12}$		
	(1)	(2)	(3)	(4)	(5)	(6)
$P_t(r_{t,t+1} < -2\sigma_{t,t+1})$	-1.08^{***}			-1.06^{***}		
(s.e.)	(0.31)			(0.21)		
$P_t(r_{t,t+3} < -2\sigma_{t,t+3})$		-0.76^{**}			-0.74^{***}	
(s.e.)		(0.31)			(0.20)	
Economic policy uncertainty			26.74^{***}			20.27^{**}
(s.e.)			(8.94)			(7.90)
No. obs.	248	248	248	236	236	236
Adj. R^2	0.11	0.05	0.09	0.22	0.12	0.10

Table 7: **Financial Intermediary Leverage and Higher-Moment Risk.** This table reports the slope coefficients of regression of measures of higher-moment risk onto the intermediary leverage of He, Kelly, and Manela (2016). Panel A reports the slope coefficient for for univariate regressions. Panel B reports the slope coefficients for regressions where we include variance on the right hand side. We report standard errors in parentheses and significance as: * when $p < 0.1$, ** when $p < 0.05$, and *** when $p < 0.01$. We correct standard errors for autocorrelation using Newey and West (1987).

Panel A: Loadings of Higher-Moment Risk Measures on Leverage

Horizon	$P_t(r_{t,t+T} < -2\sigma_{t,t+T})$	HRI	Var (%)	Skew	Kurt	Hskew	Hkurt
Month	-0.26	0.15	0.11^{***}	0.05	-0.25	1.57	-8.56
(s.e.)	(0.29)	(0.29)	(0.03)	(0.07)	(0.33)	(3.96)	(38.24)
Quarter	0.03	-0.03	0.23^{**}	0.01	-0.01	0.24	-0.01
(s.e.)	(0.36)	(0.50)	(0.10)	(0.07)	(0.43)	(2.21)	(17.08)

Panel B: Loadings of Higher-Moment Risk Measures on Leverage, Controlling for Variance

Horizon	$P(r_{t,T} < -2\sigma_{t,T})$	HRI	Var (%)	Skew	Kurt	Hskew	Hkurt
Month	0.33^{***}	-0.49^{***}	—	-0.08^{*}	0.69^{***}	-7.03^{***}	75.32^{***}
(s.e.)	(0.12)	(0.18)	—	(0.05)	(0.24)	(2.38)	(24.40)
Quarter	0.56^{***}	0.75^{***}	—	-0.11^{***}	0.56^{***}	-3.55^{***}	24.34^{***}
(s.e.)	(0.16)	(0.20)	—	(0.04)	(0.10)	(0.92)	(5.90)

Table 8: **Market Liquidity, Funding Liquidity, and Higher-Moment Risks.** This table reports the slope coefficients of regression of measures of higher-moment risk onto market liquidity (Panel A) and funding liquidity (Panel B). We run regressions at with both monthly and quarterly horizon measures. We use the value-weighted bid-ask spread of S&P 500 constituents as a measure for market illiquidity. We use the TED spread as a measure for funding illiquidity. We report standard errors in parentheses and significance as: * when $p < 0.1$, ** when $p < 0.05$, and *** when $p < 0.01$. We correct standard errors for autocorrelation using Newey and West (1987).

Panel A: Loadings of Higher-Moment Risk Measures on Bid Ask Spread

Horizon	$P(r_{t,T} < -2\sigma_{t,T})$	HRI	Var (%)	Skew	Kurt	Hskew	Hkurt
Month	-1.06^{***}	-0.75^{***}	0.17^{***}	0.29^{***}	-1.60^{***}	14.82^{***}	-139.10^{***}
(s.e.)	(0.10)	(0.11)	(0.05)	(0.04)	(0.24)	(2.39)	(27.40)
Quarter	-0.96^{***}	-1.08^{***}	0.39^{***}	0.26^{***}	-0.96^{***}	6.90^{***}	-41.96^{***}
(s.e.)	(0.10)	(0.15)	(0.11)	(0.03)	(0.15)	(0.93)	(6.94)

Panel B: Loadings of Higher-Moment Risk Measures on TED Spread

Horizon	$P(r_{t,T} < -2\sigma_{t,T})$	HRI	Var (%)	Skew	Kurt	Hskew	Hkurt
Month	-1.08^{***}	-0.93	0.25^{**}	0.22	-2.45^{**}	21.98	-234.73
(s.e.)	(0.41)	(0.61)	(0.11)	(0.13)	(1.06)	(14.30)	(423.76)
Quarter	-0.78^{**}	-1.03	0.55^{**}	0.10	-1.21^{**}	7.20	-55.52
(s.e.)	(0.36)	(0.63)	(0.22)	(0.12)	(0.47)	(4.39)	(53.90)

Table 9: **"Bubble" Characteristics and Higher-Moment Risks.** This table reports the results of regressions of the higher-moment risk index onto the following characteristics: acceleration, CAPE, dividend-price ratio, cay, turnover, and issuance. We report standard errors in parentheses and significance as: * when $p < 0.1$, ** when $p < 0.05$, and *** when $p < 0.01$. We correct standard errors for autocorrelation using Newey and West (1987).

	Dependent variable: Monthly HRI								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Acceleration	−3.18								1.36^{*}
(s.e.)	(2.66)								(0.71)
CAPE		0.01							-0.09^{**}
(s.e.)		(0.07)							(0.04)
Dividend-price ratio			1.79						−61.33
(s.e.)			(82.14)						(69.12)
Cay				-36.56^{***}					-28.01^{***}
(s.e.)				(7.89)					(7.30)
Turnover					-2.06^{***}	1.38			−0.17
(s.e.)					(0.67)	(0.86)			(1.45)
Turnover$\times r_{t-24,t}$						6.00^{***}			−4.27
(s.e.)						(2.01)			(4.84)
Issuance							−0.86	−0.92	4.10^{*}
(s.e.)							(2.22)	(1.98)	(2.09)
Issuance$\times r_{t-24,t}$								-15.77^{***}	−9.59
(s.e.)								(4.84)	(6.65)
$r_{t-24,t}$						0.28		6.31^{***}	5.54^{*}
(s.e.)						(0.69)		(1.46)	(2.89)
No. obs.	264	264	260	83	240	240	240	240	80
Adj. R^2	0.03	−0.00	−0.00	0.34	0.07	0.30	−0.00	0.19	0.44

Table 10: **Correlations Between S&P 500 Moments Under Risk-Neutrality** This table reports pairwise correlations between monthly risk-neutral S&P 500 moments: variance (Var), skewness (Skew), kurtosis (Kurt), hyperskewness (Hskew), and hyperkurtosis (Hkurt). We report 95% bootstrapped confidence bounds in brackets.

	Er	Var	−Skew	Kurt	−Hskew	Hkurt
Er	1	0.99	−0.46	−0.50	−0.48	−0.41
		[0.99,1]	[−0.56,−0.37]	[−0.57,−0.46]	[−0.55,−0.43]	[−0.48,−0.37]
Var		1	−0.52	−0.54	−0.51	−0.44
			[−0.60,−0.43]	[−0.60,−0.50]	[−0.58,−0.47]	[−0.51,−0.40]
−Skew			1	0.80	0.78	0.66
				[0.76,0.84]	[0.74,0.82]	[0.60,0.72]
Kurt				1	0.97	0.93
					[0.95,0.98]	[0.90,0.95]
−Hskew					1	0.98
						[0.97,0.98]
Hkurt						1

Table 11: **Principal Components of Higher-Moment Risks Under Risk-Neutrality.** We estimate the four principal components (PC) spanning the space of monthly risk-neutral skewness (Skew), kurtosis (Kurt), hyperskewness (Hskew), and hyperkurtosis (Hkurt). The last column of Panel A shows that the first principal component ($PC1^{RN}$) explains 95% of the variation in monthly risk neutral higher order moments.

	$Skew^{RN}$	$Kurt^{RN}$	$Hskew^{RN}$	$Hkurt^{RN}$	Variation explained
$PC1^{RN}$ eigenvector	−0.48	0.51	−0.51	0.50	95%
$PC2^{RN}$ eigenvector	0.78	−0.08	−0.27	0.57	5%
$PC3^{RN}$ eigenvector	−0.41	−0.76	0.10	0.49	0%
$PC4^{RN}$ eigenvector	−0.00	−0.38	−0.81	−0.44	0%
$PC1^{RN}$ correlation	−0.94	0.99	−0.99	0.97	

Appendix A Ex Ante Physical Moments and Risk-Neutral Pricing

Ex ante physical moments and risk-neutral pricing

Using equation (6) we can represent physical ex ante moments in terms of asset prices:

$$E_t[R_{t,T}^i] = \frac{E_t^*[R_{t,T}^{i+\gamma}]}{E_t^*[R_{t,T}^{\gamma}]}$$

for $i \in \{1, ..., 6\}$. These asset prices can be used to estimate ex ante physical moments by expanding the standardized moment formula in equation (7). We estimate kurtosis, hyperskewness, and hyperkurtosis in the following way:

$$\text{Kurtosis}_{t,T} = \frac{E_t[R_{t,T}^4] - 3E_t[R_{t,T}]^4 + 6E_t[R_{t,T}]^2 E_t[R_{t,T}^2] - 4E_t[R_{t,T}]E_t[R_{t,T}^3]}{(E_t[R_{t,T}^2] - E_t[R_{t,T}]^2)^2}$$

$$\text{Hyperskewness}_{t,T} = \frac{E_t[R_{t,T}^5] + 4E_t[R_{t,T}]^5 + 10E_t[R_{t,T}]^2 E_t[R_{t,T}^3] - 10E_t[R_{t,T}]^3 E_t[R_{t,T}^2] - 5E_t[R_{t,T}]E_t[R_{t,T}^4]}{(E_t[R_{t,T}^2] - E_t[R_{t,T}]^2)^{5/2}}$$

$$\text{Hyperkurtosis}_{t,T} = \frac{E_t[R_{t,T}^6] - 5E_t[R_{t,T}]^6 + 15E_t[R_{t,T}]^4 E_t[R_{t,T}^2] - 20E_t[R_{t,T}]^3 E_t[R_{t,T}^3] + 15E_t[R_{t,T}]^2 E_t[R_{t,T}^4]}{(E_t[R_{t,T}^2] - E_t[R_{t,T}]^2)^3}$$

$$- \frac{6E_t[R_{t,T}]E_t[R_{t,T}^5]}{(E_t[R_{t,T}^2] - E_t[R_{t,T}]^2)^3}$$